GOVERNMENT SPENDING AND CONSUMPTION-HOURS PREFERENCES

J. DAVID LOPEZ-SALIDO (BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM) (*)
AND PAU RABANAL (CAIXA D’ESTALVIS I PENSIONS DE BARCELONA)

Abstract. In this paper we present two extensions that have been largely omitted in the recent literature on estimation of DSGE models. First, we pay special attention to different forms of complementarity between consumption and hours affecting the households preferences. Second, we allow for the presence of a fraction of non-Ricardian households –i.e. that have limited access to financial markets. These two features are needed to allow for different channels of transmission of fiscal policy shocks, and its simultaneous introduction poses a well-known identification problem to estimate intertemporal substitution models of the business cycle. We show that exogenous changes in government transfers are crucial to distinguish between the two sources of comovements of consumption and hours in response to government spending shocks. We find that allowing for consumption-hours complementarity leads to a smaller estimate of the fraction of non-Ricardian households than under separability. The main conclusion from the estimated models is that private consumption increases after a government spending shock, when either nonseparability, non-Ricardian behavior, or both, are introduced in the model.

JEL Classification: E32, E62.

Keywords: Non-Ricardian Consumers, Fiscal Policy, Bayesian Estimation.

Date: May 19, 2008. (*) Corresponding author. Research Department, Caixa d’Estalvis i Pensions de Barcelona, Avinguda Diagonal 621-629, Torre 1, Planta 6, 08028 Barcelona, Spain. Email: prabanal@lacaixa.es.

We thank Alejandro Justiniano, Jean-Philippe Laforte, and Juan Rubio-Ramirez for useful feedback on some of the issues discussed in this paper. The opinions expressed here are solely those of the authors and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System, or of Caixa d’Estalvis i Pensions de Barcelona (“la Caixa”).
1. INTRODUCTION

The construction and estimation of Dynamic Stochastic General Equilibrium (DSGE) models for monetary policy analysis has witnessed an impressive development in recent years (see, for instance, Smets and Wouters (2003), and Christiano, Eichenbaum, and Evans (2005)). In this paper, we focus on the effects of fiscal policy on private consumption and output in an estimated DSGE model, and we present two extensions largely omitted in this literature but that were at the heart of the development of intertemporal substitution models of the business cycle during the eighties.¹

These two directions are clearly rooted as prior beliefs from the micro-empirical literature on consumption (see, e.g. the survey by Attanasio (1999)), and are key to generating different effects (either positive or negative) of fiscal policy on private consumption (for VAR evidence on the effects of fiscal policy see Blanchard and Perotti (2002) and Perotti (2004)). First, we pay special attention to the potential importance of complementarity between consumption and hours, thus we do not restrict the preferences to be separable between consumption and hours. Second, we extend the representative agent model in a simple way. In addition to the Ricardian intertemporal optimizing agents, there is a fraction of non-Ricardian households, i.e. these will not be engage in bonds market trading, and hence consume their wage income period-by-period. Mankiw (2000) provides a recent survey on this issue, while Souleles (1999) and Parker (1999) provide evidence of violation of the permanent income hypothesis using data on tax refunds.

Our main results are as follows: First, we show that once we allow for consumption-hours complementarity, we only estimate a small, yet stable over time, and significant fraction of non-Ricardian households. Otherwise, with separable preferences, our estimates for rule-of-thumb behavior become as large as 50 percent.² Second, in a variety of model specifications, government spending crowds in private consumption. Third, the marginal data density favors a heterogeneous agent model with non-separable preferences and enough variation in labor demand due to changes in firm’s markups. Finally, our DSGE-based estimated small government spending shocks lead to a positive comovement between consumption and hours, even when we estimate a representative agent model with non-separable preferences. Moreover, we show

¹Hall (1980, 1986), Barro (1981), and Barro and King (1984) constitute classical references. The first two papers crucially emphasized the role of exogenous shifters in government expending as a critical identification device of the intertemporal approach to the business cycles. See also Baxter and King (1993) and, more specially, King (1990) for a complete list of references and a excellent exposition of the preferences side of the model.

²Papers estimating DSGE models with non-Ricardian behavior and separable preferences, with a focus on the effects of fiscal policy include Coenen and Straub (2005), and Forni, Monteforte, and Sessa (2006).
that ‘Big-War-Time’ events, as in Ramey (2006), do not have an impact on our estimated
government spending shocks.

We reach these conclusions by embedding the two previous considerations into a DSGE New
Keynesian (New Neoclassical) model that we estimate using Bayesian methods. An obvious
advantage of the Bayesian approach is that information on the model’s parameters can be
introduced via the prior distribution. Adopting a general equilibrium full information perspec-
tive, and estimating the model’s parameters taking into account the cross-equation restrictions
implied by the solution of the model allows to better understand which forces are at play. For
instance, the presence of rule-of-thumb consumers or nonseparable preferences is not enough to
explain an increase of private consumption after a government spending shock. As stressed by
Galí, López-Salido and Vallés (2006), the monetary policy rule and the degree of price stickiness
also play a critical role in shaping the response of consumption.

The main results obtained in this paper represent a departure from two recent papers that
represent polar cases regarding both the effects and the mechanisms explaining the subsequent
transmission of government spending shocks. On the one hand, Ramey (2006) emphasizes how
many special features the model must contain to explain the rise in consumption. In addition,
Ramey’s paper has made explicit the possibility that Structural VAR-based government spend-
ing shocks reflect (anticipated) responses to War-time dummies (‘large shocks’) associated to
specific and infrequent expansions in military spending (the so-called narrative approach, see
Ramey and Shapiro (1988)). On the other hand, Bilbiie and Straub (2006) emphasize, using
a Bayesian estimation strategy the need to account for a substantial increase in the share of
agents participating in asset markets after mid-eighties as well as a change in the monetary
policy rule (from passive to active).

There are two reasons that the set of results in Bilbiie and Straub (2006) are not war-
anted. First, their model imposes zero complementarity between consumption and hours in
the preferences. This limits the identification of the existence of non-Ricardian effects from the
consumption-hours complementarity (e.g. Basu and Kimball (2002)). Second, we show that
one way to disentangle between these two effects consists on using the information contained
in government transfers. These two changes overturn their conclusions, so that the role played
by non-Ricardian consumers is clearly overstated and there are no symptoms of substantial
subsample instability in the fraction of agents with limited access to asset markets. Regarding
Ramey (2006), we follow the recommendation in Fernandez-Villaverde et al. (2006) to verify
the usefulness of VARs by estimating, by Bayesian methods, the deep parameters of a full spec-
ified model that allows for consumption-hours non-separability. Hence, this departure from
the recent studies that employ VARs allows to make use of the cross-parameter restrictions implied by the model. Our estimated, DSGE-based government spending shocks always lead to a positive comovement between consumption and hours.

The outline of the paper is as follows. In Section 2 we lay out the implications for the intertemporal Euler equation of the existence of complementarity between consumption and hours as well as a fraction of agents with limited asset market participation. We pay special attention to the role of exogenous variations in government transfers to distinguish between the two circumstances. In section 3 we describe the data and the estimation strategy, with special attention to the choice of our priors. Section 4 presents our benchmark estimates for the period 1954:I-2004:IV. In section 5 we test the robustness of the results against two extensions: preferences that imply a negligible wealth effect in labor supply; and deviations from perfect competition in the labor market. Section 6 quantifies that pitfalls identifying the parameter of preferences and the reduced form parameter capturing deviations from Ricardian behavior without using exogenous variation in government transfers. We also explore the subsample stability of our results. Section 7, compares the Bayesian estimated government spending shocks and the Ramey-Shapiro War-Time dummies. Finally, we recap the main conclusions. Several Appendices contain additional details on the model equations and the parameter estimates.

2. The Model

In this section we only discuss in details our two departing assumptions –preferences versus asset market participation– that matter for the join dynamics of consumption and hours, and so for the identification of the effects of government spending shocks on those variables using likelihood-based methods. We later discuss how to extend the model in two directions by considering time non-separable preferences with a weak short-run wealth effect on the labor supply as well as deviations from a perfectly competitive labor market.\textsuperscript{3,4}

The rest of the model that we use to study the effects of government spending on consumption and hours is a widely used among both business cycle researchers and policy makers. It is a medium scale New Keynesian macro model along the lines of Christiano, Eichenbaum, and Evans (2005) – CEE, hereandafter– and Smets and Wouters (2003) –SW, hereandafter–. Apart from the previously mentioned considerations three additional features are incorporated in the model: the presence of nominal rigidities, the existence of adjustment costs to investment, and variable capacity utilization.

\textsuperscript{3}See Jaimovich and Rebelo (2006) and Hall (2006) for a recent application of these preferences.

\textsuperscript{4}Since the model is a fairly standard New Keynesian one, we leave its detailed presentation for the Appendix.
In the empirical part below we are concerned about explaining the behavior of eight macroeconomic variables. Hence, to avoid singularity issues in the likelihood function, the model has eight shocks. We focus on the analysis of three fiscal shocks, government spending, transfers, and tax (rate) shocks. In addition, we include three technology shocks, a monetary policy shock and price markup shocks. Of the three technology shocks, one is stationary (neutral) and the other two have a unit root (investment-specific and labor-augmenting).

2.1. Households. There are two types of households in the economy. A fraction \(1 - \lambda\) of infinitely-lived households whose conditional welfare at a given time \(t\) is defined as the discounted sum of expected period utility:

\[
W_t = E_t \sum_{k=0}^{\infty} \frac{\beta^k}{1-\sigma} \left[ (C_{t+k}^0)^a (1 - N_{t+k}^0)^{1-a} \right]^{1-\sigma}
\]  

(2.1)

where \(\beta\) is the discount factor; and \(C_t^0\) and \(N_t^0\) denotes the household’s total consumption and hours, respectively. We refer to these households as the optimizers. The parameter \(\sigma \geq 0\) captures risk-aversion/intertemporal-substitution attitudes of these households; and the constant inside the kernel, \(0 < a < 1\), reflects the relative importance of consumption and leisure in the utility function, and it is usually helps to pin down a steady state value of per-capita-hours.

These preferences are part of the family of kernels that satisfy balanced growth conditions (e.g. King, Plosser, Rebelo (1988)). In particular, if \(\sigma > 1\) (more curved than log), then \(U_{CN} > 0\), such that an increase in hours worked increases the marginal utility of consumption, and hence consumption\(^5\). If \(\sigma < 1\), \(U_{CN} < 0\) then raising hours worked decreases the marginal utility of consumption. Under separable preferences, which is typically the case of most estimated DSGE models (for instance, SW (2003)), these effects are not present and \(U_{CN} = 0\).

Optimizing households can save either by investing in capital goods, or in a bond that costs one dollar and that pays a gross interest rate of \(R_t\) in the following period. Optimizing households also make capital utilization decisions.\(^6\)

For unmodeled reasons (e.g. myopia, limited access to financial markets, or –continuously–binding borrowing constraints), we assume that the remaining fraction of households, \(\lambda\), solve, at each period, a static problem, max \(\frac{1}{1-\sigma} [(C_t^r)^a (1 - N_t^r)^{1-a}]^{1-\sigma}\), subject to the zero-savings constraint:

\[
C_t^r = (1 - \tau_t) \frac{W_t N_t^r}{P_t} + T_t
\]  

(2.2)

\(^5\)To see this point, note that \(U_{CN} = \frac{\partial^2 U}{\partial C_t \partial N_t} = a(\sigma - 1)(1 - a)C_t^a(1-\sigma)^{-1}(1 - N_t)^{(1-a)(1-\sigma)^{-1}}\).

\(^6\)We assume that labor, capital and profits are taxed at the same rate (e.g. Schmitt-Grohé and Uribe, 2006). A complete description of the budget constraint and the equilibrium conditions for these households is in the Appendix.
We call these households, *rule-of-thumb consumers*. Both types of households, optimizers and rule of thumbers, pay taxes, where $\tau_t$ denotes income tax rate, and receive $T_t$ net transfers from the government.

As shown in the Appendix, under the previous assumptions it is possible to characterize the aggregate labor supply and the intertemporal allocation of consumption and hours worked through the following two equations:

$$\frac{C_t}{1-N_t} \cdot \frac{1-a}{a} = (1-\tau_t) \frac{W_t}{P_t}$$  \hspace{1cm} (2.3)

$$E_{t}\beta R_t \left\{ \frac{P_t}{P_{t+1}} \left[ \frac{C_{t+1} \left[ 1-\left( \frac{(1-a)\lambda}{1-N_{t+1}} \right) \right]^{-\alpha a T_{t+1}}}{C_t \left[ 1-\left( \frac{(1-a)\lambda}{1-N_t} \right) \right]^{-\alpha a T_t}} \right]^{\sigma} \left[ \left( \frac{C_{t+1} \left( 1-N_{t+1} \right)}{C_t \left( 1-N_t \right)} \right)^{-(1-a)(1-\sigma)} \right] \right\} = 1$$  \hspace{1cm} (2.4)

Several comments are in order. First, as can be seen from expression (2.3) the utility cost of supplying labor increases at the same rate as the real wage, such that hours are stationary around the growth path of consumption and real wages, for these class of time-nonseparable preferences. Second, a noticeable aspect of equation (2.4) is that only depends on aggregate measures of consumption and hours worked, and hence it is a convenient form of avoiding to keep track of the relative consumption across households, between those who can reoptimize and those who cannot.

Third, the consumption Euler equation (2.4) can be reduced to more familiar formulations under alternative simplifying assumptions. For instance, if utility is logarithmic ($\sigma \to 1$) and there are no rule of thumb consumers ($\lambda \to 0$), then we go back to an Euler equation where consumption growth depends on the real rate of interest, i.e. the model implies that the marginal utility of consumption does not depend upon the hours worked.

Finally, and more importantly for the purpose of this paper, the presence of the transfers, $T_t$, delivers a strong case for identification in our subsequent empirical analysis. In particular, under the assumption of $T_t = 0$, then the two models –under non-separable preferences ($\lambda = 0$, $\sigma \neq 1$) or with separable preferences and limited asset participation ($\lambda \neq 0$ and $\sigma = 1$)– become observational equivalent. This latter aspect as been extensively emphasized in the consumption literature (see e.g. Attanasio (1999) for a survey) and more recently rescued by Basu and Kimball (2002), in the context of GMM estimation, yet there is no likelihood based-analysis inside a fully specified DSGE model.

2.1.1. *Log-linear Approximation*. To fix some ideas and to relate our model with the literature, we now present a linear approximation of the previous consumption-hours Euler equation
around the (steady state) balanced growth path. We use lower case variables to denote deviations from steady-state values of stationary variables, and lower case variables with a tilde those variables that have been normalized by the combination of the levels of technology to make them stationary (see Appendix for details). Hence, after some algebra, the resulting log-linear approximation to the aggregate Euler equation (2.4) can be written as follows,

\[ \tilde{\sigma}_t \Delta \tilde{c}_{t+1} = (r_t - E_t \Delta p_{t+1}) + (\vartheta_N + \vartheta_\lambda) E_t \Delta n_{t+1} + \vartheta_T E_t \Delta \tilde{t}_{t+1} \]  

(2.5)

where \( \tilde{\sigma} = \sigma + (1 - a)(1 - \sigma) \), \( \vartheta_N = \frac{N}{1 - N} \frac{1 - a}{\alpha} (\tilde{\sigma} - 1) \), \( \vartheta_\lambda = \frac{\sigma_{c_e}}{1 - \vartheta_e} \frac{N}{1 - N} \), \( \vartheta_c = \lambda \left( \frac{1 - a}{1 - N} \right) \), \( \vartheta_T = \frac{\sigma a \lambda}{\gamma_c (1 - \vartheta_e)} \), and \( \gamma_c \) is the steady-state consumption-output ratio.

Interestingly, for this family of preferences, the presence of rule of thumb consumers (i.e., \( \lambda \)) does not affect the intertemporal response of aggregate consumption to changes in the real interest rate, as opposed to other papers in the literature (see e.g. Galí, López-Salido and Vallés, 2005; Cavallo, 2002; and Bilbiie, 2006).

On the right hand side of the Euler equation (2.5) there are two additional variables that have been (critically) omitted in the recent papers that emphasized the role of ‘intertemporal disturbances’ as driving forces of business cycle, see e.g. Primiceri et al. (2006) and Christiano and Davis (2006). Allowing for consumption-hours complementarity and non-Ricardian consumers will clearly matter for the identification of such a disturbances and constitutes a clear avenue for further research.

In the absence of non-Ricardian consumers (i.e., \( \lambda = 0 \)) the previous equation (2.5) corresponds to the one estimated by Basu and Kimball (2002). To see this point more clearly, it can be rearranged as follows:

\[ E_t (\Delta \tilde{c}_{t+1} - \kappa \Delta n_{t+1}) = \frac{1}{\sigma} (r_t - E_t \Delta p_{t+1} - \kappa \Delta n_{t+1}) \]  

(2.6)

where, as shown in the Appendix, the parameter \( \kappa \) corresponds to the steady state after tax labor income-consumption expenditure ratio. Alternatively, this parameter can also be related to preferences as follows: \( \kappa \equiv \frac{1 - \sigma}{\sigma} \varphi \), with \( \varphi \equiv \frac{N}{1 - N} \) representing the inverse of the Frisch labor supply elasticity. This leaves open two possible empirical approaches to estimated expression (2.6). On the hand, as discussed by Basu and Kimball (2002) it is possible to use steady state information to set a value for \( \kappa \) (i.e. for the U.S. economy this leads to a value for this parameter in the range of 0.8) to pin down the elasticity of intertemporal substitution (\( \frac{1}{\sigma} \)). On the other, it also possible to estimate the parameter \( \varphi \), given a value for \( a \), as well as \( \frac{1}{\sigma} \).

Finally, from comparing equations (2.5) and (2.6), it is clear that introducing a new parameter to be estimated (\( \lambda \)) to (2.6) will result in identification problems, since (2.5) without transfers (\( t_t = 0 \)) is likely to deliver several combinations of \( \sigma \) and \( \lambda \) for which the numerical value of
\( \theta_N + \theta_\lambda \) is roughly the same. It is critical to use information on net transfers, to be able to estimate \( \sigma \) and \( \lambda \), and avoid the identification problems put forth by Canova and Sala (2006). This is the approach we take in the empirical part of the paper.

2.2. Government and Fiscal Policy. To close the remarks on the model that we estimate in the paper, we now describe the government budget constraint and the fiscal policy rules.

Each period, the government consumes \( \tilde{G}_t \) units of the composite good, and \( \tilde{y}_t \) will represent the government spending in deviations from steady state, and normalized by steady state output, i.e. \( \tilde{y}_t = (\tilde{G}_t - \bar{G})/\bar{Y} \). We assume that the variable \( \tilde{y}_t \) is exogenous and that it follows the following log-linear first-order autoregressive process

\[
\tilde{y}_t = \rho_y \tilde{y}_{t-1} + \varepsilon^g_t
\]

where \( \rho_y \in (0,1) \) and \( \varepsilon^g_t \) is an i.i.d. government spending shock.

Like government consumption, transfers are assumed to be exogenous and also follow an AR(1) in logs:

\[
\tilde{t}_t = \rho_t \tilde{t}_{t-1} + \varepsilon^t_t
\]

where \( \tilde{t}_t = \frac{T_t - T}{Y} \), the parameter \( \rho_t \in (0,1) \), and \( \varepsilon^t_t \) is an i.i.d. shock.

The fiscal authority covers deficits by issuing one-period, nominally risk-free bonds, \( B_t \). A log-linear approximation to the period-by-period government budget constraint and the economy-wide resource constraint are given by

\[
\bar{\tau} (\tau_t + \tilde{y}_t) + \beta \tilde{b}_t = \tilde{b}_{t-1} + \tilde{y}_t + \tilde{t}_t - (\varepsilon^a_t + \frac{1}{1-\alpha} \varepsilon^v_t)
\]

and

\[
\tilde{y}_t = \gamma_c \tilde{c}_t + \gamma_i \tilde{i}_t + \alpha \frac{1}{1+\mu_p} (1-\bar{\tau}) u_t.
\]

where, as shown in the Appendix, \( \varepsilon^a_t \) and \( \varepsilon^v_t \) are innovations to the two unit root technology shocks of the model.

The government levies labor, capital, and profit income taxes, and we assume that the marginal tax rate follow the rule,

\[
\tau_t = \rho_\tau \tau_{t-1} + (1-\rho_\tau) \phi_b \tilde{b}_{t-1} + \varepsilon^{tax}_t
\]

where \( \tilde{b}_t = (\tilde{B}_t/P_{t-1} - \bar{B}/P)/\bar{Y} \) and \( \varepsilon^{tax}_t \) is an i.i.d. tax shock. In this case, we follow the recent paper by Schmitt-Grohe and Uribe (2006) and we write the rule in terms of the marginal tax

\[\text{We have used the fact that total tax revenue is } \tilde{T}_t = \tau_t \tilde{Y}_t, \text{ and hence } \tilde{t}_t = \frac{\tilde{T}_t - T}{Y} = \bar{\tau} \tilde{y}_t + \bar{\tau} \tau_t = \bar{\tau} (\tilde{y}_t + \tau_t).\]
rate that depends linearly on its own lag and past log deviations of government liabilities.\footnote{This fiscal policy rule is different than the one considered by Galí, López-Salido and Vallés (2006) and Coenen and Straub (2005). In those papers, the fiscal policy rule is written in terms of lump-sum tax revenues, and it reacts to the current levels of government spending and debt.}

While this rule does not come from a maximizing welfare analysis, Schmitt-Grohe and Uribe (2006) specify this family of rules as to approximate optimal rules. The rationale is as follows: first, the government increases spending to stimulate economic activity. However, over the medium term it ensures fiscal sustainability by raising the marginal tax rate if necessary. This is achieved by having a positive response of the marginal tax rate with respect to the level of debt. In order to have short-term expansionary effects of government spending policy, the tax rate reacts after several quarters. Note that, since government spending is expansionary, the tax base (and tax revenues) also increase with a government spending shock (something that does not necessarily happen under lump-sum taxation).\footnote{We want to isolate the effect of government spending shocks by restricting the reaction of tax rates to be zero for several quarters. If we allow the tax rate to react immediately, then we would be mixing the effects of the two main fiscal policy tools (tax rates and government spending). This is why we also depart from Schmitt-Grohe and Uribe (2006) and do not consider the case where the tax rate reacts to the output gap.}

3. Data and Estimation Strategy

We estimate the parameters of the model using Bayesian methods, and analyze the implications of the model regarding the effects of government spending shocks and the contribution of the latter to the comovements between consumption and hours worked. The use of Bayesian methods to estimate dynamic stochastic general equilibrium models has increased over the recent years, in a variety of contexts (see for instance the survey provided by An and Schorfheide, 2006). In this section we briefly outline the estimation procedure, as well as the data sources.

3.1. The Data. We estimate the model using eight observable variables: per capita output growth ($\Delta y_t$), per capita consumption growth ($\Delta c_t$), per capita hours ($n_t$), government spending and transfers growth, in percent of potential output ($\Delta g_t$ and $\Delta t_t$, respectively), government deficit as percent of potential output ($def_t$), nominal interest rates ($r_t$) and inflation ($\Delta p_t$). We demean all these variables.

For inflation and nominal interest rates we have direct counterparts for the data in the model. Because of the presence of unit roots in the two technology processes, real variables are nonstationary in levels but stationary in first differences in the model. Hence, we use the first difference operator to detrend the data in a model-consistent way. In general, for the non-stationary variables, the following relationship hold between its first differences in the data...
and in the model:

$$\Delta \tilde{z}_t = \tilde{z}_t - \tilde{z}_{t-1} + \varepsilon_t^a + \frac{\alpha}{1 - \alpha} \varepsilon_t^v$$

for $\tilde{z}_t = \{\tilde{y}_t, \tilde{c}_t, \tilde{g}_t, \tilde{t}_t\}$, and where $\alpha$ is the elasticity of output to capital. The definition for the government deficit as a percent of potential output is:

$$def_t = \bar{\tau}(\tau_t + \tilde{y}_t) - \tilde{g}_t - \tilde{t}_t$$

The sample period is 1954:I to 2004:IV. Our data sources are as follows: we obtain real output and consumption from the National Income and Product Accounts (NIPA). For consistency with previous empirical work, our measure of government spending is total government spending in real terms, as percent of potential GDP (see, Galí, López-Salido and Vallés (2006) and the references therein). We also checked our results with non-military government spending. Our measure of inflation is the GDP deflator, while the relevant nominal interest rate is the three month T-bill. Our measure of the deficit is the difference between government savings and investment, as percent of potential GDP. Finally our measure of hours is the NFBS hours per capita (although we also checked for robustness using employment). Population is defined as Civilian Noninstitutional Population, 16 years and over. The measure of net transfers is total transfer payments less transfer receipts.\(^{10}\)

As is well know from Bayes’ rule, the posterior distribution of the parameters is proportional to the product of the prior distribution of the parameters and the likelihood function of the data. In order to implement the Bayesian estimation method, we need to be able to evaluate numerically the prior and the likelihood function. Then, we use the Metropolis-Hastings algorithm to obtain random draws from the posterior distribution, from which we obtain the relevant moments of the posterior distribution of the parameters, as well as posterior impulse responses.

\(^{10}\)As in Galí, López-Salido and Vallés (2006), the series were drawn from Estima’s USECON database. These include government (Federal + State + Local) consumption and gross investment expenditures (GH), nominal and real gross domestic product (GDP and GDPH), a measure of aggregate hours obtained by multiplying total civilian employment (LE) by weekly average hours in manufacturing (LRMANUA), nonfarm business hours (LXNFH), the real compensation per hour in the nonfarm business sector (LXNFR), consumption of nondurable goods and services (CNH+CSH), transfer payments (GETFP) and receipts (GRTFR), and the CBO estimate of potential GDP (GDPPOTHQ). All quantity variables are in log levels, and normalized by the size of the civilian population over 16 years old (LNN). Our deficit measure corresponds to gross government investment (GFDI+GFNI+GSI) minus gross government savings (obtained from the FRED-II database). The resulting variable, expressed in nominal terms was normalized by the lagged trend nominal GDP (GDPPOTQ). We use the 3-month T-bill rate (FTB3) as the relevant nominal interest rate at quarterly frequency.
3.2. The Likelihood Function. As shown in the Appendix we can write a log-linear approximation to the non-linear DSGE model. We collect the linearized equilibrium conditions and we write the system in the following state space form:

\[ A(\Theta) E_t X_{t+1} = B(\Theta) X_t + C(\Theta) X_{t-1} + D(\Theta) S_t, \]

\[ S_t = N(\Theta) S_{t-1} + \varepsilon_t, \quad E(\varepsilon_t \varepsilon_t') = \Sigma(\Theta). \]

where \( \Theta \) denote the vector of parameters that describe preferences, technology, the monetary and fiscal policy rules and the shocks of the model, \( X_t \) be the vector of all endogenous variables, \( S_t \) be the vector of state variables, and \( \varepsilon_t \) be the vector of innovations. We use standard solution methods for linear models with rational expectations (Uhlig, 1999) to write the law of motion in state-space form and the Kalman filter to evaluate the likelihood of the eight observable variables \( d_t = (\Delta y_t, \Delta c_t, n_t, \Delta g_t, \Delta t, \Delta f_t, r_t, \Delta p_t)' \). We denote by \( L \left( \{d_t\}_{t=1}^T | \Theta \right) \) the likelihood function of \( \{d_t\}_{t=1}^T \).

3.3. The Choice of Priors. We denote by \( \Pi(\Theta) \) the prior distribution of the model’s parameters. We present the list of the structural parameters and its associated prior distribution in Table 1. These priors are assumed to be independent across parameters, and are based upon existing research. In the model, the parameter \( \tilde{\sigma} \) is related to risk aversion attitudes as well as the inverse of the intertemporal substitution. Following recent research by Chetty (2006), that discusses measures of risk aversion when hours are included in preferences, we set the prior mean to \( \tilde{\sigma} = 2 \), although we allow for substantial uncertainty around this value (see, for instance, the discussion contained in Hall (2006)). Following most of the business cycle literature we set the prior mean of the inverse Frisch labor supply elasticity, \( \varphi \), to one. Following the micro-evidence literature reviewed by Mankiw (2000) we set the prior means of the weight of rule-of-thumb households \( \lambda \) to 0.3, which is in the lower limit of the range of estimated values. This value is also in line with the prior mean used in the empirical analysis by Bilbiie and Straub (2006).

The prior mean of the fraction of firms that keep their prices unchanged, \( \theta_p \), is set to 0.5, and the fraction of backward looking price setters, \( \omega_p \), is set to 0.3. These two values are in the lower range of the estimates obtained from the New Keynesian Phillips curve literature (see, e.g. Galí, Gertler, and López-Salido (2001) and Eichenbaum and Fisher (2005)) and the available micro evidence (e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2006)). Overall, we also set prior standard deviations that are large enough to incorporate the uncertainty about those parameters in the existing literature.

The policy parameters are chosen as follows. We set the prior mean of the response of the monetary authority to inflation, \( \phi_\pi \), to 1.5, and the prior mean of the smoothing interest
rates parameter, $\rho_c$ equals 0.5. These values commonly used in empirical Taylor rules (see e.g. Woodford, 2001). Finally, the mean of the $\phi_b$ is set equal to 0.1 which is in line with the estimated parameter by Bohn (1998).

Finally, as described in the last rows of Table 1, we consider a uniform distribution for both the autocorrelation and the standard deviation of the model’s shocks. Uniform priors make sense if there is no pertinent research on the question of interest and if there are no theoretical reasons to favor one hypothesis. In addition, by using uniform priors we try to avoid that by imposing too much structure on the priors of the shocks, we might end up affecting the estimates of the model’s structural parameters. Finally, as described in the last rows of Table 1, we consider a uniform distribution for both the autocorrelation and the standard deviation of the model’s shocks. Uniform priors make sense if there is no pertinent research on the question of interest and if there are no theoretical reasons to favor one hypothesis. In addition, by using uniform priors we try to avoid that by imposing too much structure on the priors of the shocks, we might end up affecting the estimates of the model’s structural parameters. Finally, as described in the last rows of Table 1, we consider a uniform distribution for both the autocorrelation and the standard deviation of the model’s shocks. Uniform priors make sense if there is no pertinent research on the question of interest and if there are no theoretical reasons to favor one hypothesis. In addition, by using uniform priors we try to avoid that by imposing too much structure on the priors of the shocks, we might end up affecting the estimates of the model’s structural parameters. Finally, as described in the last rows of Table 1, we consider a uniform distribution for both the autocorrelation and the standard deviation of the model’s shocks. Uniform priors make sense if there is no pertinent research on the question of interest and if there are no theoretical reasons to favor one hypothesis. In addition, by using uniform priors we try to avoid that by imposing too much structure on the priors of the shocks, we might end up affecting the estimates of the model’s structural parameters.

In addition, several parameters of the model are fixed instead of being estimated. Those include, the discount factor that we set $\beta = 0.995$, the capital share of output, that we set to $\alpha = 0.33$. The parameters that govern investment dynamics are set to $\eta = 1/2.48$, while the elasticity of capacity utilization to the real rate of return is set to $\psi = 1$. These parameter values are close to those reported by CEE (2005) and Altig et al. (2005), and are not estimated here because we do not use data on investment expenditures. The steady state government consumption-to-output ratio is set to $\gamma_g = 0.2$, as well as the average tax rate, $\bar{\tau}$. These values are pretty conventional in the literature, and for alternative reasonable parameter values, the main results of the paper do not change.

11A thorough discussion of prior choice for the shocks of DSGE models can be found in Del Negro and Schorfheide (2006).
Table 1. Prior Distributions of the Model’s Parameters

<table>
<thead>
<tr>
<th>Definition</th>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Risk Aversion</td>
<td>$\tilde{\sigma}$</td>
<td>Gamma</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>Inverse of Labor Supply Elasticity</td>
<td>$\varphi$</td>
<td>Normal</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>Calvo Lottery</td>
<td>$\theta_p$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.20</td>
</tr>
<tr>
<td>Fraction on Non-Optimizer Price Setters</td>
<td>$\omega_p$</td>
<td>Beta</td>
<td>0.3</td>
<td>0.10</td>
</tr>
<tr>
<td>Fraction of Non-Ricardian Consumers</td>
<td>$\lambda$</td>
<td>Beta</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Fiscal Policy Rule</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response of Tax Rates to Debt</td>
<td>$\phi_b$</td>
<td>Normal</td>
<td>0.1</td>
<td>0.025</td>
</tr>
<tr>
<td>Tax Smoothing</td>
<td>$\rho_t$</td>
<td>Beta</td>
<td>0.6</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>Monetary Policy Rule</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response to Inflation</td>
<td>$\gamma_\pi$</td>
<td>Normal</td>
<td>1.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Interest Rate Smoothing</td>
<td>$\rho_r$</td>
<td>Uniform</td>
<td>0.5</td>
<td>0.28</td>
</tr>
<tr>
<td>AR Coefficients of Shocks</td>
<td>$\rho's$</td>
<td>Uniform</td>
<td>0.5</td>
<td>0.28</td>
</tr>
<tr>
<td>Std. Deviation of Shocks</td>
<td>$\sigma's$</td>
<td>Uniform</td>
<td>0.125</td>
<td>0.07</td>
</tr>
</tbody>
</table>

3.4. **Drawing from the Posterior.** From Bayes rule, we obtain the posterior distribution of the parameters as follows:

$$p(\Theta|\{d_t\}_{t=1}^T) \propto \mathcal{L}(\{d_t\}_{t=1}^T | \Theta) \Pi(\Theta)$$

The posterior density function is proportional to the product of the likelihood function and the prior joint density function of $\Theta$. Given our priors and the likelihood function implied by the state-space solution to the model, we are not able to obtain a closed-form solution for the posterior distributions. However, we are able to evaluate both expressions numerically. We use the random walk Metropolis-Hastings algorithm, to obtain a draw of size 100,000 from $p(\Theta|\{d_t\}_{t=1}^T, m)$. We start with an initial guess of the mode using optimization algorithms, and then run an initial chain (burn-in phase) of 25,000 draws, using the Cholesky decomposition of the Hessian of the posterior evaluated at the mode to obtain the new proposed value for the vector of parameters, $\Theta$. We use this draw to estimate the moments of the posterior distribution of the parameters, and to obtain posterior moments for the impulse responses of the endogenous variables.
4. Baseline Estimation Results

4.1. Parameter Estimates. Table 2 describes the main results under alternative specifications. We present the mean posterior for a selected group of parameters of interest, as well as the 90 percent confidence interval which appears in brackets.\footnote{The posterior distributions for the remaining parameters are available from the authors on request.}

We begin by estimating a model with rule-of-thumb consumers and nonseparable preferences. We label such a model "Heterogeneous Agents" model. Moreover, we impose that the parameter $\varphi = 1$, which comes from assuming that the steady-state fraction of hours devoted to work is $\frac{1}{2}$. In this case, the posterior mean for the elasticity of intertemporal substitution, $\tilde{\sigma}$, is slightly larger than 2. Note that a value of 1, which would be consistent with separable preferences, is clearly outside the 90 percent confidence interval. The proportion of rule-of-thumb households, $\lambda$, has a posterior mean of 0.15. Notice that this value is different than the prior mean, and it is important to note that the parameter is precisely estimated, with the 90 percent confidence interval between 0.13 and 0.17. The fiscal policy rule implies a high tax rate inertia, and a mild response of tax rates to an increase of 1 percent of the debt-to-GDP ratio of 0.07. While these parameters come from estimating the rule without using actual data on tax rates or government debt, they do seem reasonable because they imply a highly persistent tax rate, something we observe in the data. The parameters on the Taylor rule are somewhat similar to what has been obtained in the literature, however, the interest rate smoothing parameter is on the low side, with a posterior mean of 0.5.

In the second column of Table 2 we present the results of a model where the elasticity of labor supply is estimated, rather than calibrated, according to the prior distribution proposed in Table 1. In this case, we find evidence that the parameter $\varphi$ is slightly larger than one, but not much larger: the 90 percent confidence interval is 1.09 to 1.38. This implies a labor supply elasticity between 0.72 and 0.91, in line with the value recently used by Hall (2006). The parameter $\lambda$ declines to a value of 0.10, while the rest of parameter estimates do not change significantly. Notwithstanding, the Bayes factor favors the previous model where the parameter $\varphi$ was calibrated to one using steady state information.

Next, we estimate the model without rule-of-thumb consumers but with non-separable preferences (i.e. the specification in Basu and Kimball, 2002). That is, we restrict the model to have $\lambda = 0$. Note that, according to equation (2.5), imposing such restriction implies that transfers have no effect in consumption. However, transfers still enter the set of observable variables and are estimated to follow an AR (1) process. If we dropped transfers from the set of observable variables, we would not be able to compare models using the Bayes factor.
In this case, the main change is on the elasticity of intertemporal substitution, \( \tilde{\sigma} \), whose posterior mean increases to 2.81. This implies an estimated elasticity of intertemporal substitution of 0.36, slightly below the preferred estimates obtained by Basu and Kimball (2002) using limited information (GMM) methods. The rest of the parameters remain similar to the one previously estimated, with the difference that the response of tax rates to the lagged level of debt reduces, and so is the persistence of government spending shocks. We also report the marginal likelihood of the data for each model. The Bayes factor, defined as the ratio of marginal likelihoods between two different models, tells the researcher how much she would update her priors on which model is the true one after observing the data. In this case the (log) Bayes factor favors the model with rule of thumb consumers by 88. This means that a researcher should have a prior over the model without rule-of-thumb consumers which is \( \exp(88) \) times larger to the model with rule-of-thumb consumers to assign a higher posterior probability to the model without rule-of-thumb consumers being the true model, after observing the data. By any measure, \( \exp(88) \) is a very large number, suggesting that model fit improves with rule-of-thumb consumers.

Next, we proceed to estimate the model with separable preferences and rule-of-thumb consumers, which is the model used by Galí, López-Salido and Vallés (2006) and estimated by Coenen and Straub (2005). We refer to this model as the "Separable Preferences Model". In this case, we obtain a larger estimate of \( \lambda \), of almost 0.4, while the rest of parameter estimates are numerically very similar to those obtained under the Basu-Kimball preferences. The Bayes factor also suggests that the two models (Separable Preferences/Basu Kimball) are not distinguishable, and they are both inferior to the model with both features at the same time. In addition, we believe that this estimation provides a nice example of how imposing a specific type of preferences (separable versus nonseparable) can have dramatic consequences for the interpretation of a structural parameter, in this case the fraction of "rule-of-thumb" consumers. However, we want to stress that both features are necessary: contrary to Bilbiie and Straub (2006), the relative poorer performance of the separable preferences model could be due to the restrictive relationship it imposes between interest rates and the fraction of non-Ricardian consumers.
Table 2. Posterior Distribution. Baseline Estimates

<table>
<thead>
<tr>
<th></th>
<th>Heterogeneous Agents</th>
<th>Non Separable Basu-Kimball</th>
<th>Separable Preferences</th>
<th>NKK</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{\sigma} )</td>
<td>2.25 (2.02 - 2.49)</td>
<td>2.81 (2.60 - 3.02)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.15 (0.13 - 0.17)</td>
<td>0.0</td>
<td>0.39 (0.38 - 0.41)</td>
<td>0.0</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>0.83 (0.82 - 0.85)</td>
<td>0.81 (0.79 - 0.82)</td>
<td>0.80 (0.79 - 0.81)</td>
<td>0.70</td>
</tr>
<tr>
<td>( \rho_t )</td>
<td>0.98 (0.98 - 0.98)</td>
<td>0.93 (0.90 - 0.95)</td>
<td>0.98 (0.97 - 0.98)</td>
<td>0.96</td>
</tr>
<tr>
<td>( \phi_b )</td>
<td>0.07 (0.03 - 0.09)</td>
<td>0.04 (0.03 - 0.04)</td>
<td>0.05 (0.03 - 0.07)</td>
<td>0.08</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>0.98 (0.98 - 0.98)</td>
<td>0.92 (0.90 - 0.93)</td>
<td>0.98 (0.97 - 0.98)</td>
<td>0.98</td>
</tr>
<tr>
<td>( \gamma_\pi )</td>
<td>1.32 (1.27 - 1.37)</td>
<td>1.27 (1.23 - 1.30)</td>
<td>1.28 (1.27 - 1.30)</td>
<td>1.39</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>0.47 (0.41 - 0.52)</td>
<td>0.42 (0.36 - 0.48)</td>
<td>0.45 (0.44 - 0.46)</td>
<td>0.22</td>
</tr>
<tr>
<td>( \theta_p )</td>
<td>0.74 (0.71 - 0.76)</td>
<td>0.71 (0.69 - 0.74)</td>
<td>0.80 (0.78 - 0.82)</td>
<td>0.82</td>
</tr>
<tr>
<td>( \omega_p )</td>
<td>0.76 (0.72 - 0.80)</td>
<td>0.79 (0.77 - 0.82)</td>
<td>0.63 (0.59 - 0.67)</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Log L 5145.1 5101.3 5057.4 5053.3 4712.5

Finally, for comparison purposes, we also estimated a model without rule of thumb consumers, and with separable preferences, which would be in the spirit of most DSGE models estimated using minimum distance (CEE, 2005) or Bayesian methods (SW, 2003; Rabanal, 2007). We labelled this model "NKK" (New Keynesian model with capital).\(^{13}\) The first result to note is that the marginal likelihood is much lower, with the log Bayes factor with respect to the model with all the features of 432.6. Hence, introducing into the model features that might change the response of private consumption and hours to shocks greatly improves model fit. Also, the probability of the Calvo lottery increases to 0.82, which implies an average duration of price contracts of 6 quarters, as opposed to a posterior mean of roughly 4 quarters in the baseline case.\(^{14}\) In addition, the fraction of rule-of-thumb price setters decreases to about 0.6,

\(^{13}\)We cannot label this model as "CEE" or "SW" because in those papers wages are sticky à la Calvo, while in the present paper we assume they are flexible. Hence the label "New Keynesian model with capital" sounds more appropriate.

\(^{14}\)These estimates are similar to Rabanal and Rubio-Ramírez (2005) in a model with separable preferences, representative agents, and no capital.
while it was around 0.8 in all other models. This estimated larger fraction of backward-looking price setters could be due to the fact that the price mark-up shock is iid. Possibly, if we had allowed for an autocorrelated price mark-up shock the backward looking behavior parameter would have been smaller, but we chose not to do so to avoid overparametrizing the model.\textsuperscript{15}

4.2. Posterior Impulse Responses. In Figure 1 we examine what are the effects on consumption, investment, output, and hours to a normalized government spending shock in each of the previously estimated models. We first describe the responses of the models on the first column where we compare the responses of the two “Heterogeneous Agents” models (with fixed $\varphi$, and with separable preferences), with two representative agent models, under non-separable (Basu-Kimball) and separable (NKK) preferences, respectively.\textsuperscript{16} Following a persistent increase in government spending, private consumption raises when either nonseparable preferences or rule-of-thumb consumers, or both, are introduced in the model.\textsuperscript{17} The impulse responses under either rule-of-thumb consumers or nonseparability are numerically very similar, while the effect is larger when the two features are in place. The effects are not very long lived, although it takes four quarters for the effect to turn negative. When the two features are switched off, then we obtain the expected result that government spending crowds out consumption (the model corresponding to the last column in Table 2).

In the Bayesian analysis the posterior distributions of the model’s parameters, as well as the posterior impulse response functions, depend not only on the obtained data but also on the prior probabilities over the model’s parameters. To understand how much our chosen priors affect the impulse response functions, in the first top four panels of Figure 2 we compare the prior mean response of consumption across the models with our posterior mean estimates. As can be seen there are substantial differences between prior and posterior indicating that the likelihood values of the model given the data provide substantial information on the parameters of interest. In the Heterogeneous Agents model, the prior implies a larger impact response of consumption to a government spending shock, that changes signs after one period. The estimated posterior implies a smaller impact but more persistence. For the Basu-Kimball model, the prior implies a small, positive response of consumption, that turns negative immediately. The posterior

\textsuperscript{15}Del Negro and Schorfheide (2006) find that it is difficult to answer the question of backward looking behavior versus autocorrelated price mark-up shocks in the New Keynesian model.

\textsuperscript{16}The impulse responses of the model with estimated $\varphi$ are very similar to those where $\varphi = 1$, and hence are not shown.

\textsuperscript{17}Coenen and Straub (2006) find that the response of private consumption to a government spending shock is always negative, by estimating a model with separable preferences and rule-of-thumb consumers for the euro area.
assigns a higher impact and persistence. The most interesting result happens in the Separable Preferences case: the prior mean implies that consumption would decline and remain always negative after a government spending shock. On the contrary, the parameter estimates push the posterior impulse response to positive territory. This is achieved by having a higher posterior mean of $\lambda$ of 0.4. Finally, as expected, in the NKK model both prior and posterior impulse responses display a negative response of consumption, although the posterior mean is smaller than the prior mean.

As noticed from the impulse responses of the Figure 1, there is not much variation in the response of consumption across models (except for the NKK model, they look very much alike), thus supporting the overall robustness of an increase of private consumption in response to government spending shocks. This is true under different model assumptions, which is possible due to the fact that it comes as a general equilibrium result. To see this, in Figures 3 and 4 we display the contour plot of the impact effect of a government spending shock on consumption as we vary the two coefficients of the price equations, $\theta_p$ and $\omega_p$, the elasticity of intertemporal substitution ($\tilde{\sigma}$) and the degree of rule-of-thumb behavior ($\lambda$). Consumption is crowded out in
response to government spending shocks for low values of these two parameters. If, instead, \( \theta_p \) and \( \omega_p \) are large then consumption increases in response to a government spending shock, given the induced shift in the labor demand equation associated with a certain amount of price stickiness. On the contrary, at the current parameter estimates, as we approach price flexibility (southwest corner of the plot, \( \theta_p, \omega_p \to 0 \)), then nonseparability or the presence of rule-of-thumb consumers would not be enough to explain an increase in government spending.\(^{18}\)

In Figure 4, we can see that even a small fraction of rule-of-thumb consumers is needed to generate a positive increase of private consumption to government spending, of around 5 percent. Around this cut-off point, the response of consumption to government spending becomes less responsive to changes in \( \tilde{\sigma} \); on the other hand for moderate levels in the degree of rule-of-thumb behavior, of around 20 percent, imply much more responsiveness of consumption to a government spending shock.

\(^{18}\)Linneman (2005) explores the possibility of generating an increase of private consumption after a government spending shock in an RBC model with nonseparable preferences.
5. Extensions

We now analyze two deviations from the previous (baseline) model to assess the robustness of the results. We begin by studying a model with nonseparable preferences that imposes no wealth effect on labor supply. Subsequently, we consider a version of the model that allows for deviations from perfect competition in the labor market. We will use the same data to obtain
the posterior means of the relevant parameters, so we will be able to compare the marginal data densities of these extensions with the baseline heterogeneous model.

5.1. No Wealth Effect. These preferences correspond to the ones analyzed by Greenwood, Hercowitz, and Huffman (1988). This element will allow consumption to respond to an increase in hours worked, while real wages do not inherit fluctuations in consumption in response to a positive government spending shock. Hence, we assume the following period utility:

$$U^i(C^i_t, N^i_t) = \frac{1}{1 - \sigma} \left[ C^i_t - \left( \frac{N^i_t}{1 + \varphi} \right)^{1+\varphi} \right]^{1-\sigma}$$

(5.1)

for \(i = o, r\). Similar preferences have been recently considered by Jaimovich and Rebelo (2006) and Hall (2006). For each period of time, \(t\), the kernel inside the brackets governs the marginal rate of substitution between consumption and hours worked, and the parameter \(\varphi\) is related to the inverse of the Frisch labor supply elasticity (see below). The parameter \(\sigma\) controls the concavity, and so it is related to risk aversion (intertemporal substitution attitudes.)

In the Appendix we show that, under this specification, the log-linear approximation of the aggregate Euler equation becomes,

$$\xi E_t \Delta c_{t+1} = \frac{1}{\sigma} (r_t - E_t \Delta p_{t+1}) - (1 - \xi) (1 + \varphi) E_t \Delta n_{t+1} + \frac{\lambda \xi}{\gamma_e} E_t \Delta \bar{t}_{t+1}$$

(5.2)

where \(\phi = \frac{1 + \varphi \lambda}{1 + \varphi}\), and \(\xi = \frac{1}{1 - \phi \varphi}\). Notice that since \(\phi\) depends on the fraction of the rule of thumb consumers, so does \(\xi\). In addition, since \(\xi > 1\), \(1 - \xi\) will be always negative. Hence, this preferences, everything else being equal, imply that there will be a positive comovement between consumption growth and hours growth.

An interesting feature of this specification is that, differently to the previous model, the presence of rule of thumb consumers (i.e., \(\lambda\)) does affect the intertemporal response of aggregate consumption to changes in the real interest rate. In particular, the response of expected consumption to changes in the real interest rates depends upon risk aversion, the labor supply elasticity, and the fraction of rule of thumb consumers (i.e. \(\frac{1}{\sigma \xi}\)). Most interestingly, and contrary to the model with separable preferences, the presence of non-Ricardian consumers

\[19\] For different motives, recently Hall (2006) and Jaimovich and Rebelo (2006) have emphasized the importance of small income effects to explain the comovements between consumption, hours, and employment, as well as to generate booms in response to expectations of higher future total factor productivity. Earlier applications to these preferences are Correia et al. (1994) in a small open economy model, and Christiano et al. (1997) to study liquidity effects models.

\[20\] The specification considered by Hall (2006) is slightly more general than expression (5.1).
will reduce the elasticity of intertemporal substitution, but it will never revert the sign of the response of consumption to real interest rates (e.g. Bilbiie, 2006 and Bilbiie and Straub, 2006)).

5.2. Imperfectly Competitive Labor Markets. We now extend the baseline heterogeneous agent model along the lines discussed by Galí, López-Salido and Vallés (2006). In particular, we assume that the wage is set by a union, hours are determined by firms’ labor demand, and (2.3) does not apply. Under the assumption of imperfectly competitive labor markets, we implicitly assume that the resulting wage markup is sufficiently high (and fluctuations sufficiently small) that the inequalities \( \frac{W_t}{r_t} > \frac{a}{1-a} \frac{C_t^j}{1-N_t^j} \) for \( j = r, o \) are satisfied at all times. Each firm decides how much labor to hire (given the wage), and allocates labor demand uniformly across households, independently of their type. Accordingly, we will have \( N_t^r = N_t^o \) for all \( t \). These conditions guarantee that both type of households will be willing to meet firms’ labor demand at the prevailing wage.

As shown in the Appendix a log-linear approximation to the aggregate Euler equation of consumption is given by the following expression:

\[
E_t \Delta \tilde{c}_{t+1} = \frac{1}{\sigma} (r_t - E_t \Delta p_{t+1}) + \\
\kappa (1 - \frac{1}{\sigma}) E_t \Delta n_{t+1} + \frac{\lambda}{1 - \lambda} (1 + \varphi) E_t \Delta n_{t+1} + \frac{\lambda}{1 - \lambda} \gamma_c E_t \Delta \tilde{c}_{t+1} \tag{5.3}
\]

which resembles previous expression (2.5) but differs in the expression of the parameters affecting the effects of hours and transfers on the expected growth of consumption.

5.3. Results. Table 3 shows the main results of the two extensions described in this section. The most important result is that the model that assumes Imperfect Labor Markets is the one that ranks highest in terms of marginal likelihood, and, as we discuss later, it implies an increase of consumption to government spending, but we need to rely on a relatively high \( \lambda \), implying that slightly less than 20 percent of households cannot smooth consumption intertemporally. For the case of GHH preferences, despite the fact that the estimated fraction of rule-of-thumb consumers is less than 10 percent, the response of consumption to government spending shocks is also positive. This response of consumption is mainly associated with a very high estimates of the degree of curvature of the consumption-hours kernel. The rest of the parameters are in line with the ones presented in the Table 2.

The plots in the second column of the Figure 1 compare the impulse response functions of the two models estimated in this section and the baseline heterogeneous model. As can be seen they are broadly similar. Consumption and hours increase, and investment falls. Nevertheless, both the increase in consumption and the windfall in investment are less pronounced in the
model without wealth effect, i.e., the one corresponding to the GHH preferences. In this case, the implicit effect of real interest rates to consumption growth would be around $\frac{1}{\sigma_x} = 0.1$, much lower than the intertemporal elasticity of substitution of $\frac{1}{\sigma} = 0.5$ implied by the imperfect labor market model. The two panels of the last row in Figure 2 confirms that there are substantial differences between priors and posterior means, again supporting that the data (the likelihood), given the model, provide substantial information on the parameters of interest, specially for the imperfect labor market model.

Finally, and most interestingly, from the comparison of the marginal data densities for these two models and the one of the baseline heterogeneous model in Table 2 it follows that data favor the GHH model relative to the benchmark. The imperfect labor market model gives an even higher marginal data density than both the GHH model and the benchmark one. Therefore, if we had to choose one model between the six estimated so far, this would be a model with nonseparable preferences, rule-of-thumb consumers, and an imperfect labor market.

<table>
<thead>
<tr>
<th>Table 3. Posterior Distribution. Extensions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GHH Preferences</strong></td>
</tr>
<tr>
<td>$\widetilde{\sigma}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
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<td></td>
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<td>$\rho_t$</td>
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<td>$\phi_b$</td>
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<td>$\rho_r$</td>
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<tr>
<td>$\theta_p$</td>
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<td></td>
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<td>$\omega_p$</td>
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<td>Log L</td>
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</table>

Note: For the GHH model, we estimated and report $\sigma$.

5.4. Responses to a Monetary and a Neutral Technology Shock. We also undertook, for completeness, the exercise of plotting the impulse responses to a monetary and to a neutral technology shock. To save space, these figures are placed in the Appendix. The most interesting
result for the monetary policy shock is the reaction under GHH preferences is quite different from all other models: the impact effect on consumption and output is smaller than in the other cases, but it certainly displays more persistence: it takes about 8 quarters for these two variables to go back to their steady-state values. In all other cases, the propagation mechanism is similar, and does not display much persistence: following a contractionary monetary policy shock, output and consumption decline, and go back to their steady state values pretty much after 4 periods. This is a consequence of not having introduced habit formation in our preferences, and hence consumption and output do not display a hump-shaped response nor much persistence. In addition, the impact effect is larger for the model with separable preferences and no rule-of-thumb consumers, which is a direct consequence of having the largest elasticity of intertemporal substitution of all models (which is one, while in all other cases the elasticity is less than one).

Regarding the impulse response of the neutral technology shock, the models that display different behavior are the Heterogeneous Agents and the NKK models. In all cases, hours decline after a neutral technology shock, consistent with the findings of Galí and Rabanal (2005). Because of nonseparabilities and the presence of rule-of-thumb consumers, the positive correlation between hours growth and consumption growth is maximized in the heterogeneous agents model: as a result, while in all the other models the response of consumption is positive or close to zero, it becomes negative for the heterogeneous agents model. On the other hand, because it incorporates no direct effects of hours growth on consumption growth, the NKK implies a strong positive response of consumption after a neutral technology shock.

6. TWO ISSUES

6.1. Pitfalls Identifying \( \delta \) and \( \lambda \). Identification of the model’s parameters has become an important issue in the literature that uses likelihood-based methods to estimate general equilibrium models. The likelihood function is evaluated using the Kalman filter and the state-space representation of the law of motion of the model, which in turn is obtained by solving a linear system of equations with rational expectations. In practice, it can be impossible to know if all parameters of the model are identified, and as Canova and Sala (2006) have shown, it could well be that the likelihood function does not depend on some "structural" parameters, after solving the model using algorithms such as Blanchard and Kahn (1980) or Uhlig (1999). Another option is that combinations of parameters deliver the same value for the likelihood function.

As we have discussed previously, the model without transfers delivers an Euler equation for consumption that makes models with rule-of-thumb consumers and non-separable preferences observationally equivalent. In this subsection, we reestimate the model with heterogeneous...
agents and the Basu and Kimball (2002) model after removing net transfers from the set of observable variables. We report the estimates for $\tilde{\sigma}$ and $\lambda$ in Table 4. The implications for the model with rule-of-thumb behavior are quite important: in the baseline model, without transfers a researcher would conclude that the fraction of rule of thumb consumers is 0.35, instead of 0.15, while the estimate of $\tilde{\sigma}$ would be 1.38 instead of 2.25. In addition, without transfers as an observed variable, the posterior mean of $\lambda$ is quite similar to the prior mean, although the standard deviation shrinks considerably. To assess whether the presence of an imperfect labor market matters for the previous result, we also report in Table 4 the same experiment for that particular model. As can be seen the previous finding holds, but quantitatively it is less noticeable than in the baseline case. When we reestimate the model with separable preferences, the estimate for $\lambda$ is even higher, implying that roughly one half of consumers cannot smooth consumption intertemporally, a fraction originally suggested in the work of Campbell and Mankiw (1989).

Finally, the parameter estimates change marginally in the Basu-Kimball model, and to save space we only report $\tilde{\sigma}$, but the rest of parameters display small numerical differences, and are available upon request. Hence, introducing transfers as an observable variable does indeed seem to help to identify $\lambda$. While the parameter estimate is smaller than what is typically suggested in the literature and the prior mean, it is precisely estimated, and the Bayes factor favors the model with rule of thumb consumers over the one without them.

<table>
<thead>
<tr>
<th>Table 4. The role of Transfers</th>
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<tbody>
<tr>
<td>Heterogeneous Agent</td>
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<tr>
<td><strong>Transfers</strong></td>
</tr>
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<td>$\tilde{\sigma}$</td>
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<tr>
<td>$\lambda$</td>
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<tr>
<td></td>
</tr>
<tr>
<td>Separable Preferences</td>
</tr>
<tr>
<td><strong>Transfers</strong></td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
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<td></td>
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<tr>
<td>$\lambda$</td>
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</table>
Overall, our results provides a substantially lower estimate for $\lambda$ than Campbell and Mankiw (1989), but are rather consistent with the microevidence provided and summarized by Attanasio and Browning (1995) and Attanasio and Weber (1995) and the recent aggregate evidence by Basu and Kimball (2002). The former papers show that aggregate estimates of the Euler equation results are driven by aggregation problems, demographics, and complementarity between consumption and hours in the preferences. This later point was forcefully emphasized by Basu and Kimball (2002).

6.2. Subsample Instability. The aim of this section is to try to shed some preliminary evidence of two separate issues recently emphasized by Perotti (2004). First, the extend to which the effects of government spending on consumption (and so the output multiplier) are smaller in the Volcker-Greenspan period. And second, wether we are able to find a decline in the variance of the fiscal shocks and in their transmission mechanism. Thus, we estimate the previous models over two samples, corresponding to the pre-Volcker (1954:Q1-1979:IV) and the Volcker-Greenspan periods (1982:I-2004:IV).

In Figure 5 we compare the normalized impulse responses of consumption and output to a government spending shock for three estimated models across subsamples. Two comments are in order. First, we find a positive response of consumption across subsamples. Second, there is some evidence that the responses are somewhat smaller in the second half of the sample. This is particularly clear for the baseline model with heterogeneous agents and non-separable preferences and the model of imperfect labor markets. However, as shown in the bottom panel of Figure 5, if we consider the model with separable preferences such an inference vanishes.

We now will try to disentangle how the previous responses depend upon the transmission mechanism and the persistence properties of the shocks. Table 5.A. presents the posterior estimates of three set of parameters. The first two rows present the preference parameter, $\tilde{\sigma}$, and the fraction of non-Ricardian consumers, $\lambda$. These are two crucial parameters for the Euler equation in the model. The next two rows are related to changes in the price stickiness and the amount of backward lookingness affecting the Phillips curve (i.e. parameters, $\theta_p$ and $\omega_p$). The last two rows decompose the unconditional variance of government spending in the first order autocorrelation coefficient, $\rho_g$, and the conditional variance of the innovations, $\sigma_g$. Finally, we compare the evidence for two different models with or without assuming separability between consumption and hours. These results help in understanding the differences in the responses of consumption and output presented in the previous Figure 5.

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21The rest of the posterior estimates are presented in the Appendix.
We find that, in the “Heterogeneous Agents” model, variations of the posterior distribution of the estimated risk aversion attitudes as well as in fraction of non-Ricardian consumers explain the changes in the Euler equation. Interestingly, none of these changes alter the sign of the slope of the consumption Euler equation relative to the interest rates, but increase the consumption response to real interest rates as well as to hours variations. At the same time, we find some supporting evidence for a small increase in the degree of price stickiness which helps in generating variations in the labor demand that support a higher response of wages and so of consumption to the government spending shock. We should also stress that there are significant differences in the autocorrelation of the government spending process, which has become more persistent in the later period. The implied larger wealth effect is likely to have changed the response of consumption to a government spending shock.

To understand the role of asset market participation, and its possible time variation, we also estimate the model with separable preferences across subsamples: unlike the Heterogeneous Agents model, assuming separability between consumption and hours barely alters the response
of consumption and output, as can be seen from the estimates in the last two columns of Table 5.A. If any, we find lower differences across subsamples regarding the fraction of non-Ricardian consumers and less effects on the parameters of the Phillips curve.

To further check whether this is a robust feature, in Table 5.B. we present the subsample posterior estimates of the same parameters but for the two extensions considered in the paper, namely the consideration of an imperfect labor market or GHH preferences. The results for the model with imperfect labor markets tend to support a quite remarkable stability across subsamples, while there is more time variation under GHH preferences. In all cases, therefore, the tendency is to have a smaller elasticity of intertemporal substitution, and a slightly larger fraction of rule-of-thumb consumers. The standard deviation of the innovation to a government spending shock does not change. Hence, although for different reasons, our results seem to be in line with Canova’s (2006) –he imposes zero complementarity between consumption and hours– that changes in the intertemporal elasticity of substitution explain why the transmission mechanism of monetary policy (and other) shocks has changed.

**Table 5.A. Posterior Distribution. Subsample Stability**

<table>
<thead>
<tr>
<th></th>
<th>Heterogeneous</th>
<th>Separate</th>
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<tbody>
<tr>
<td></td>
<td>Agent</td>
<td>Preferences</td>
</tr>
<tr>
<td></td>
<td>54-79</td>
<td>82-04</td>
</tr>
<tr>
<td>( \tilde{\sigma} )</td>
<td>3.31 (2.94 - 3.74)</td>
<td>2.07 (1.79 - 2.34)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.04 (0.03 - 0.06)</td>
<td>0.13 (0.11 - 0.16)</td>
</tr>
<tr>
<td>( \theta_p )</td>
<td>0.59 (0.50 - 0.67)</td>
<td>0.69 (0.64 - 0.73)</td>
</tr>
<tr>
<td>( \omega_p )</td>
<td>0.76 (0.70 - 0.83)</td>
<td>0.87 (0.84 - 0.90)</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>0.46 (0.38 - 0.53)</td>
<td>0.84 (0.81 - 0.87)</td>
</tr>
<tr>
<td>( \sigma_g )</td>
<td>0.012 (0.010 - 0.014)</td>
<td>0.011 (0.009 - 0.012)</td>
</tr>
<tr>
<td>Log L</td>
<td>2473.9</td>
<td>2546.8</td>
</tr>
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</table>
GOVERNMENT SPENDING AND CONSUMPTION-HOURS PREFERENCES

Table 5.B. Posterior Distribution. Subsample Stability

<table>
<thead>
<tr>
<th></th>
<th>Imperfect Labor Market</th>
<th>GHH Preferences</th>
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<tr>
<td></td>
<td>54-79</td>
<td>82-04</td>
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<tr>
<td>$\tilde{\sigma}$</td>
<td>2.01</td>
<td>1.64</td>
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<tr>
<td></td>
<td>(1.76 - 2.13)</td>
<td>(1.57 - 1.70)</td>
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<tr>
<td>$\lambda$</td>
<td>0.14</td>
<td>0.16</td>
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<tr>
<td></td>
<td>(0.12 - 0.17)</td>
<td>(0.15 - 0.16)</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>0.68</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(0.66 - 0.71)</td>
<td>(0.68 - 0.71)</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>0.67</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(0.61 - 0.71)</td>
<td>(0.76 - 0.78)</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.81</td>
<td>0.84</td>
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<tr>
<td></td>
<td>(0.78 - 0.81)</td>
<td>(0.84 - 0.84)</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.013</td>
<td>0.013</td>
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<td></td>
<td>(0.012- 0.014)</td>
<td>(0.012- 0.014)</td>
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<tr>
<td>Log L</td>
<td>2477.1</td>
<td>2498.3</td>
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7. Small Government Spending Shocks and War-Time Dummies

In this section we try to answer the following question: what are the effects of the big military build-ups on our “small” government spending shocks? We label our shocks “small” as opposed to “large” military build-ups associated to specific low frequency expansions of military spending (the so-called narrative approach, see Ramey and Shapiro (1988)). In doing so we notice that there have been some discussions about the extent to which VAR-based innovations are informative about how an particular economic model respond to shocks (see, for instance, Kehoe (2006) for the debate on technology shocks). Regarding the VAR identification of “normal times” government spending shocks (e.g. Blanchard Perotti (2002) and Galí, López-Salido and Vallès (2006)), in a recent paper Ramey (2006) has made explicit the possibility that most of these shocks reflect (anticipated) responses to “large” shocks, that she proxies using War-time dummies.\footnote{The effort in distinguishing between ‘small’ and ‘big’ shocks can be traced back to Blanchard and Watson (1986) seminal paper. Interestingly, in the Discussion Summary, Blanchard “felt that the prevailing view of the profession, as sposed by Solow, seemed to be that large shocks were unique events.. ” and “the sources of small perturbations were harder to isolate.” (p. 166).}

As recently noticed by Fernandez-Villaverde et al. (2006) a natural recommendation to verify the usefulness of a VAR consists in estimating the deep parameters of a fully specified model by likelihood methods. In this paper we depart from the recent studies that employ VARs that
typically only make use of some but not all of the restrictions implied by economic theories. An obvious advantage of our strategy is that adopting a general equilibrium full information perspective, and estimating the model’s parameters taking into account the cross-equation restrictions implied by the solution of the model allows to better understand which forces are at play.

To answer this question we follow Eichenbaum (1998) and we run a simple bivariate VAR model using quarterly data from 1954:II to 2004:IV on a set of dummy variables and the estimated government spending shocks from several estimated DSGE model. We refer to \(RS_t\) as the Ramey-Shapiro dummy variables, where \(RS_t = 1\) if \(t = 1965: I, 1980: I, 2001: III\), and zero otherwise. The VAR is ordered with \(RS_t\) first and our different time series of “small” government spending shocks second. Placing the \(RS\) dummy first reflects the recent claim by Ramey (2006) that these events are prior of subsequent movements in government spending shocks – so exogenous big military build-up might affect contemporaneously our estimated shocks but are not affected by them—. The lag-length of the VAR is four lags, and we also report a +/-2.0 error confidence bands computed via 5000 Monte Carlo simulations.

In Figure 6 we plot the accumulated responses of the government spending shocks to the \(RS\) dummy for four estimated models. As can be seen, on average, the \(RS\) big-events leads to a slightly initial decrease in the government spending shocks that subsequently increase leading to a permanent positive movement. Nevertheless, such an increase is small and non-significantly different from zero. Moreover, in the case of the estimated model under GHH preferences the point estimated effects are even negative.

Finally, we check whether the average results derived from the previous VAR shadow a different pattern for each of the 3-military build ups. Hence, we re-estimate the VAR model considering only one specific event at a time. The results for the baseline model heterogeneous model are presented in Figure 7. As can be seen, around the Vietnam War and the 9/11 events there is a positive impact on the estimated small shocks, albeit non-significant. This is not the case for the aftermath of the Carter-Reagan episode where we found some initial negative

\(^{23}\)Ramey and Shapiro (1998) identify three political events that led to large, exogenous increases in military expenditures. These events, which we refer to as Ramey-Shapiro episodes, coincide roughly with the onset of the Korean War, the Vietnam War, and the Carter-Reagan defense buildup. Recently Eichenbaum and Fisher (2004) and Ramey (2006) have added a new date: 9/11. Notice that our sample period limits the use of the Korean War dummy.

\(^{24}\)The responses are very similar if we estimate a 6-lags VAR. A RATS file to replicate the results is available upon request.
**Figure 6.** Responses to Ramey-Shapiro (War Time) Dummies of the Bayesian ML Estimated Government Spending Shocks. Alternative Models. Two variables VAR, sample period 1954:II-2004:IV. Point estimates and the 5% and 95% quantiles of the distribution of the responses obtained by 5000 Monte Carlo simulations of the residuals of the VAR. Horizontal axes represent the time horizon after the shock measured in quarters.

effects on our model-based government spending shocks. Again, after two quarters the effects became positive but non-significant.

**8. Concluding Remarks**

In this paper we present two extensions, clearly rooted as prior beliefs from the micro-empirical literature, that have been largely omitted in the recent literature on Bayesian estimation of DSGE models. We introduce these two extensions to allow for several effects of fiscal policy in a medium-scale macroeconomic model, and estimate how important they are. First, we pay special attention to different forms of complementarity between consumption and hours affecting the households preferences. Second, we allow for the presence of a fraction of non-Ricardian households —i.e. that have limited access to financial markets—. These two features pose a well-known identification problem to estimate/calibrate “intertemporal substitution” models of the business cycle, and, in particular, Euler equations. We show that exogenous changes in government transfers are crucial to distinguish between the two sources of comovements of consumption and hours in response to government spending shocks. Our main result
is that in several versions of estimated DSGE models, private consumption increases after a government spending shock. The only case where this effect does not happen is in the baseline New Keynesian model with capital, where an increase of consumption after a government spending shock is ruled out by construction. However, such a model ranks worst using standard methods to compare models in the Bayesian framework. In addition, we show that allowing for consumption-hours complementarity leads to a small-and-stable-over-time (but significant) estimated the fraction of non-Ricardian households. Interestingly, our DSGE-based government spending shocks always lead to a positive comovement between consumption and hours; and are not affected by “Big-War Time” events. We also present different robustness and subsample stability tests that support these results.

9. References


Appendix: A General Description of the Model

Households. Optimizing Consumers. Their budget constraint is given by:

\[ C_t^o + I_t^o + \frac{B_t^o}{P_t R_t} + \frac{\Psi(u_t)K_{t-1}^o}{V_t} = (1 - \tau_t)\left(\frac{W_t N_t^o}{P_t} + R_t^k u_t K_{t-1}^o + D_t^o\right) + \frac{B_{t-1}^o}{P_t} + T_t^o \]

where \( \tau_t \) is the average income tax rate, and \( T_t^o \) is the net transfers received from the government. Optimizing households make investment decisions \( (I_t^o) \), and also capital utilization decisions \( (u_t) \), for which they pay a cost \( \Psi(u_t)/V_t \) per unit of capital. The cost function has the following properties: \( \Psi(1) = 0 \), and \( \Psi''(\cdot) > 0 \). \( V_t \) represents the current state of technology to produce capital goods. We will refer to it as an investment-specific technological progress (Greenwood, Hercowitz, and Krusell (1996)), and assume that it follows a unit root process

\[ \log(V_t) = \log(V_{t-1}) + \epsilon_t^v \]

Households rent capital to firms, for which they get a rental price of \( R_t^k \). \( D_t^o \) denotes real profits. The law of motion of capital in this economy follows CEE:

\[ K_t^o = (1 - \delta)K_{t-1}^o + [1 - S(I_t^o/I_{t-1}^o)]I_t^o V_t \]

where the \( S(.) \) function has the following properties: \( \bar{S} = \bar{S}' = 0 \), and \( \bar{S}'' > 0 \). Hence, the investment-specific technology shock has a permanent effect on capital.

We denote by \( Q_t \) the shadow price of investment in terms of consumption goods. The first order conditions for the optimizing consumer’s problem are:

\[ \frac{C_t^o}{1 - N_t^o} \frac{1 - a}{a} = (1 - \tau_t) \frac{W_t}{P_t} \] (9.1)

\[ 1 = \beta R_tE_t \left\{ \frac{C_t^o U_{t+1}^o}{C_{t+1}^o U_t^o} \right\} \] (9.2)

\[ V_t Q_t \left[ 1 - S \left( \frac{I_t^o}{I_{t-1}^o} \right) - S' \left( \frac{I_t^o}{I_{t-1}^o} \right) \frac{I_t^o}{I_{t-1}^o} \right] \] (9.3)

\[ + \beta E_t \left\{ V_{t+1} Q_{t+1} \left( \frac{C_t^o U_{t+1}^o}{C_{t+1}^o U_t^o} \right) S' \left( \frac{I_{t+1}^o}{I_t^o} \right) \left( \frac{I_{t+1}^o}{I_{t-1}^o} \right)^2 \right\} = 1 \]

\[ Q_t = \beta E_t \left\{ \frac{C_t^o U_{t+1}^o}{C_{t+1}^o U_t^o} \right\} \left[ (1 - \tau_{t+1})R_{t+1}^k u_{t+1} - \frac{\Psi(u_{t+1})}{V_{t+1}} + (1 - \delta)Q_{t+1} \right] \] (9.4)

\[ (1 - \tau_t)R_t^k = \frac{\Psi'(u_t)}{V_t} \] (9.5)

where \( U_t^o = \frac{1}{1-a} [(C_t^o)^a (1 - N_t^o)^{1-a}]^{1-a} \).

Rule of Thumb Consumers.

Note that lump-sum transfers or taxes paid by rule-of-thumb households \( (T_t) \) do not differ from those of the optimizing households. The associated first order condition is given by:

\[ \frac{C_t^r}{1 - N_t^r} \frac{1 - a}{a} = (1 - \tau_t) \frac{W_t}{P_t} \] (9.6)
Aggregation. Aggregate consumption and hours are a weighted average of the corresponding variables for each consumer type. Formally:

\[ C_t \equiv \lambda C_t^\rho + (1 - \lambda) C_t^o \] (9.7)

and

\[ N_t \equiv \lambda N_t^\rho + (1 - \lambda) N_t^o \] (9.8)

Note that by taking a weighted average of (9.1) and (9.6) we obtain the following relationship between aggregate variables:

\[ \frac{C_t}{1 - N_t} \frac{1 - a}{a} = (1 - \tau_t) \frac{W_t}{P_t} \] (9.9)

This, in turn, implies that:

\[ N_t^\rho = 1 - \left( \frac{1 - a}{a} \right) \frac{C_t^\rho}{(1 - \tau_t) \frac{W_t}{P_t}} \]

and, therefore,

\[ C_t^\rho = a \left[ (1 - \tau_t) \frac{W_t}{P_t} + T_t^\rho \right] \]

so that aggregate consumption depends on aggregate variables and consumption of optimizing consumers:

\[ C_t = a\lambda \left[ (1 - \tau_t) \frac{W_t}{P_t} + T_t^\rho \right] + (1 - \lambda) C_t^o \] (9.10)

Note that by substituting (9.1) and (9.10) in the utility function of optimizing consumers, we obtain that:

\[ U_t^o = \left( \frac{1 - a}{a} \right)^{(1 - \alpha)(1 - \sigma)} \frac{C_t^o(1 - \sigma)^{1 - \sigma}}{(1 - \alpha)(1 - \sigma)^{1 - \sigma}} \left[ \frac{1 - \sigma}{\omega_t} \right] \] (9.11)

where \( \omega_t = W_t/P_t \). Using expressions (9.11), (9.9) and (9.10) into the expression (9.2), and assuming that the level of net transfers is the same across households, yields the expression (2.4) of the main text.

Firms: Final Goods Producers. We assume that the economy is populated by a continuum of monopolistically competitive firms producing differentiated intermediate goods. These goods are then used as inputs by a (perfectly competitive) firm producing a single final good. Final goods producers operate in perfect competition. They use all intermediate goods, indexed by \( i \in [0, 1] \), and aggregate them with the following technology:

\[ Y_t = \left[ \int_0^1 (Y_t^i)^{\epsilon_t} \frac{1}{\epsilon_t} \, di \right]^{\frac{\epsilon_t}{\epsilon_t - 1}} \]

where \( \epsilon_t > 1 \) is the time-varying elasticity of substitution. As a result, their demand functions for each type of good \( i \in [0, 1] \) are:

\[ Y_t^{i,d} = \left( \frac{P_t^i}{P_t} \right)^{-\epsilon_t} Y_t \] (9.12)

where the price level is given by the non-profit condition in this sector.

\[ P_t = \left[ \int_0^1 (P_t^i)^{1 - \epsilon_t} \, di \right]^{\frac{1}{1 - \epsilon_t}} \]
Intermediate Goods Producers. There is a continuum of intermediate goods producers, indexed by $i \in [0, 1]$. They all have access to the same production function:

$$Y_i^t = X_t(K_{i,t-1})^\alpha (A_t N_{i,t})^{1-\alpha}$$

(9.13)

where $\alpha$ is the elasticity of output to capital; $K_{i,t-1} = u_t K_{i,t-1}$, and firms take as given the capacity utilization decision by optimizing households.

The labor-augmenting technology shock follows a unit root process (in logs):

$$\log(A_t) = \log(A_{t-1}) + \varepsilon_t$$

The shock $X_t$ is a stationary technology shock that follows an AR(1) process in logs.

We adopt the Galí and Gertler (1999) modified Calvo model of inflation dynamics. Intermediate goods producers set prices with a Calvo-type restriction. Let $\theta_p$ denote the probability of not resetting prices in a given period. We assume that a fraction $\omega_p$ of them follow a backward-looking rule of thumb in price setting, that we detail below.

First, firms decide optimally how to use their inputs, and the following optimality conditions hold:

$$\frac{\omega_t}{R_t^k} = \frac{\alpha}{1-\alpha} \frac{K_{i,t-1} u_t}{N_t}$$

(9.14)

$$MC_t = \frac{1}{\alpha(1-\alpha)(1-\alpha)} \frac{(R_t^k)^\alpha}{X_t(A_t)^{(1-\alpha)}} (W_t)^{1-\alpha}$$

(9.15)

where $MC$ is the real marginal cost of production. Second, the Calvo-type restriction allows us to specify the evolution of the price level recursively, as follows:

$$P_t^{1-\varepsilon_t} = \theta_p P_{t-1}^{1-\varepsilon_t} + (1-\theta_p)(P_t^*)^{1-\varepsilon_t}$$

(9.16)

where $P_t^*$ denotes prices being reset at time $t$. Out of these, a fraction $\omega_p$ are set in a rule-of-thumb manner, while a fraction $1-\omega_p$ are set optimally. Hence,

$$(P^*)^{1-\varepsilon_t} = \omega_p (P_t^k)^{1-\varepsilon_t} + (1-\omega_p) (P_t^f)^{1-\varepsilon_t}$$

The optimal price is given by the following optimal condition under Calvo pricing:

$$\sum_{k=0}^{\infty} \theta_p^k \Lambda_{t,t+k} \left( \frac{P^f_t}{P_{t+k}} - MC_{t,t+k} \right) Y_{t,t+k}^{i,d} = 0$$

(9.17)

where $\Lambda_{t,t+k} = \beta^k \frac{U(C_{t,t+k}) C_{t+k}}{U(C_t) C_{t+k}}$ is the stochastic discount factor, $Y_{t,t+k}^{i,d}$ is the associated demand to the optimal price $k$ periods ahead, and $MC_{t,t+k}$ the associated real marginal cost of production.

The rule-of-thumb price setters set the following price:

$$P_t^b = \frac{P^*_t}{P_{t-1}}$$

Hence, they look at last period’s optimal prices and update them with last period’s inflation rate. As a result, rule-of-thumb price setters use information dated at $t-1$ and earlier. Another property of this rule of thumb is that there are no persistent deviations between optimal and nonoptimal behavior.

Government. The government conducts fiscal and monetary policy with two autonomous entities.
Fiscal Policy. The government consumes a fraction of the final good. The intertemporal budget constraint of the government is given by:

$$\tau_t Y_t + \frac{B_t}{P_t R_t} = G_t + T_t + \frac{B_{t-1}}{P_t}$$  \hspace{1cm} (9.18)

The government’s fiscal policy rules are defined in the main text.

Monetary Policy. An independent central bank conducts monetary policy using the nominal interest rate in response to past interest rates and current inflation, i.e.

$$R_t = \bar{R}^{1-\rho_r} R_{t-1}^{\rho_r} \{(P_t/P_{t-1})^{\gamma_*}\}^{1-\rho_r} \exp(\varepsilon^*_r)$$  \hspace{1cm} (9.19)

where \(\varepsilon^*_r\) is a Normally distributed iid shock.

Market Clearing. In equilibrium labor, intermediate and final goods markets clear. The economy wide resource constraint is given by:

$$Y_t = C_t + I_t + G_t + \Psi(u_t) V_t K_{t-1}$$  \hspace{1cm} (9.20)

Balanced Growth Path

Since we have assumed that the investment-specific and the labor-augmenting technology shocks have a unit root, we have that the following variables are non stationary: \(Y_t, I_t, \bar{C}_t, K_t, G_t, B_t/P_t, I^o_t, C^o_t, K^o_t, R^k_t, Q_t,\) and \(\omega_t.\) The remaining variables \(R_t, N_t, \tau_t, MC_t, P^*_t/P_t\) and the inflation rate \((P_t/P_{t-1})\)are stationary. We normalize all nonstationary variables by \(Z_t = A_t V_t^{\alpha/\beta}\), except for the stock of capital, which is divided by \(Z_t V_t\), and the rental rate of capital and Tobin’s \(Q,\) which are multiplied by the level of the investment specific technology shock, \(V_t.\)

The steady state of the normalized system is characterized by the following relationships. Since we assume zero inflation rate, the nominal and real interest rates are given by:

$$R = \frac{1}{\beta}$$

The real rental rate of capital is then: \((1-\bar{\tau})R^k = R - (1-\delta) = \Psi'(1),\) where the level of the investment-specific and labor augmenting technology shocks have been normalized to one. Tobin’s \(Q = 1,\) by the properties of the adjustment cost function. In the symmetric equilibrium, real marginal costs of production are \(MC = 1/(1+\mu_p).\) Therefore the capital-output ratio is given by:

$$\frac{\bar{Y}}{K} = \frac{R^k}{\alpha(MC)}$$

while investment and capital are related as follows: \(\frac{\bar{I}}{K} = \delta.\) As a result, the consumption-output ratio is given by: \(\gamma_c = 1 - \frac{\bar{K}}{\bar{Y}} \frac{\bar{I}}{K} - \frac{\bar{C}}{\bar{Y}}.\)

Next, using the aggregate labor supply expression (9.9):

$$\frac{N}{1 - N} \frac{1 - a}{a} = \frac{(1-\tau)WN}{PC}$$

Note that in the steady state: \(\frac{W}{P} = (1-\alpha)MC \frac{Y}{N},\) then we can combine the previous two expressions to get an expression of the after-tax labor income to consumption ratio (\(\kappa\)):

$$\kappa = (1-\alpha)(1-\tau) \frac{MC}{\gamma_c}$$
GOVERNMENT SPENDING AND CONSUMPTION-HOURS PREFERENCES

Notice also that, \( \kappa \equiv \frac{1-a}{a} \varphi \), where \( \varphi \equiv \frac{N}{1-N} \). Finally, note that in the steady state:

\[
\bar{\tau} - \frac{\tilde{G}}{\bar{Y}} - \frac{\tilde{T}}{\bar{Y}} = \frac{R-1}{R} \frac{\tilde{B}}{\bar{P} \bar{Y}}
\]

If a country has a positive debt-to-GDP ratio, then it needs to run primary surpluses to stabilize that ratio after interest payments. If the stock of debt is zero, then taxes equal spending plus net transfers in the steady state. This is an assumption, that we carry over the rest of the paper, implies that \( \bar{\tau} = \frac{\tilde{G}}{\bar{Y}} + \frac{\tilde{T}}{\bar{Y}} \).

Dynamics

We take a linear approximation of the system’s dynamics along the balanced growth path. We use lower case variables to denote deviations from steady-state values of stationary variables, and lower case variables with a tilde those variables that have been normalized by the combination of the levels of technology (i.e. \( \tilde{\omega}_t = \omega_t - z_t \)). The resulting linear equations are as follows. The labor supply schedule is given by:

\[
\tilde{\omega}_t - \frac{\bar{\tau}}{1-\bar{\tau}} \tau_t = \tilde{c}_t + \varphi n_t \tag{9.21}
\]

Notice that we can set \( N \) in different ways that will generate different values for the labor supply elasticity \( \varphi^{-1} \). As in CEE, the relationship between the shadow price of investment and its growth rate is given by:

\[
\eta \tilde{q}_t = (1+\beta)\tilde{r}_t - \tilde{v}_{t-1} - \beta E_t \tilde{v}_{t+1} + \varepsilon^u_t + \frac{\alpha}{1-\alpha} \varepsilon^v_t, \tag{9.22}
\]

where \( \eta = 1/S''(.) \), and \( \varepsilon^u_t \) and \( \varepsilon^v_t \) are the innovations to the permanent neutral and investment-specific technology shocks. The law of motion of capital is given by

\[
\tilde{k}_t = (1-\delta)\tilde{k}_{t-1} + \delta \tilde{r}_t - (1-\delta)(\varepsilon^u_t + \frac{1}{1-\alpha} \varepsilon^v_t) \tag{9.23}
\]

The rental rate of capital and the utilization rate are given by

\[
\psi(\tilde{r}^k_t - \frac{\bar{\tau}}{1-\bar{\tau}} \tau_t) = u_t, \tag{9.24}
\]

where \( \psi = \Psi'(1)/\Psi''(1) \). The relationship between the shadow price of capital and its rental rate is given by

\[
\tilde{q}_t = -(r_t - E_t \Delta p_{t+1}) + \partial_q E_t \tilde{q}_{t+1} + (1-\partial_q) E_t \tilde{r}^k_{t+1} - (1-\partial_q) \frac{\bar{\tau}}{1-\bar{\tau}} E_t \tau_{t+1} \tag{9.25}
\]

with \( \partial_q = (1-\delta)\beta \). The loglinear production function and optimal capital-labor ratios are given by:

\[
\tilde{y}_t = x_t + \alpha(u_t + \tilde{k}_{t-1}) + (1-\alpha)n_t - \alpha(\varepsilon^u_t + \frac{1}{1-\alpha} \varepsilon^v_t) \tag{9.26}
\]

and

\[
\tilde{\omega}_t + n_t = u_t + \tilde{k}_{t-1} + \tilde{r}^k_t - \varepsilon^u_t - \frac{1}{1-\alpha} \varepsilon^v_t
\]

Inflation dynamics is given by the following expression:

\[
\Delta p_t = \frac{\beta \theta_p}{\phi} E_t \Delta p_{t+1} + \frac{\omega_p}{\phi} \Delta p_{t-1} + (1-\omega_p) \kappa_p(\alpha \tilde{r}^k_t + (1-\alpha) \tilde{\omega}_t - x_t + \varepsilon^p_t) \tag{9.27}
\]
where $\kappa_p = \frac{(1-\theta_p)(1-\beta)}{\phi}$, $\phi = \theta_p + \omega_p(1-\theta_p(1-\beta))$, and where $\varepsilon_t^p$ can be interpreted as a price mark-up shock.

Finally, in addition to the fiscal policy rules, the log-liner monetary policy rule takes the following familiar form:

$$r_t = \rho r_{t-1} + (1-\rho)\gamma \Delta p_t + \varepsilon_t^r$$  \hspace{1cm} (9.28)

**Extensions**

Preferences without Income Effects. The labor supply and the euler equation of consumption of the optimizers take following the form:

$$(N_o^t)^\varphi = (1-\tau_t)\frac{W_t}{P_t}$$  \hspace{1cm} (9.29)

$$1 = \beta R_tE_t\left\{\left[\frac{X_{t+1}^o}{X_t^o}\right]^{-\sigma}\frac{P_t}{P_{t+1}}\right\}$$  \hspace{1cm} (9.30)

where $X_t^o = \left[C_t^o - \left(\frac{N_t^o}{1+\varphi}\right)^{1+\varphi}\right]^{-\sigma}$ represents the marginal utility of consumption. In addition, it also follows that the labor supply of the non-optimizer agent is given by

$$(N_r^t)^\varphi = (1-\tau_t)\frac{W_t}{P_t}$$  \hspace{1cm} (9.31)

Then, it follows that the aggregate labor supply is

$$(N_t)^\varphi = (1-\tau_t)\frac{W_t}{P_t}$$  \hspace{1cm} (9.32)

and that, in equilibrium, $N_t = N_r^t = N_o^t$. Notice that from expression (9.32) it follows that aggregate hours worked are not stationary, since they will rise permanently in response to a permanent increase in the real wage associated to technology shocks. We make hours stationary by introducing a trend in the utility function such that the disutility cost of supplying hours increases at the same rate as the real wage (see e.g. Jaimovich and Rebelo (2006a,b)).

Using (9.31), and (9.32) we obtain an expression linking the consumption of the rule of thumb consumers and hours worked

$$C_t^r = (N_t)^{1+\varphi} + T_t^r$$

We substitute the previous expression into the definition of aggregate consumption to obtain

$$C_t^o = \frac{1}{1-\lambda}(C_t - \lambda N_t^{1+\varphi} - \lambda T_t^r)$$

Hence, using the previous expression into the marginal utility of consumption of $X_t^o$ it follows that expression (9.30) can be written in terms of both aggregate consumption and hours worked:

$$1 = \beta R_t E_t\left\{\left[\frac{X_{t+1}}{X_t}\right]^{-\sigma}\frac{P_t}{P_{t+1}}\right\}$$  \hspace{1cm} (9.33)

\[^{26}\text{In other words, a basic justification of the potential presence of a trend in per capita hours is related to home production. For evidence supporting the non-stationarity of hours see, for instance, Galí and Rabanal (2004), Galí (2005), Fernald (2005), and Francis and Ramey (2005).}\]
where \( X_t = \left[ C_t - \phi(N_t)^{1+\varphi} - \lambda T_t \right] \), where the parameter \( \phi = \frac{1+\varphi\lambda}{(1+\varphi)} \). Taking a loglinear approximation of (9.33) delivers:

\[
\sigma E_t \Delta x_{t+1} = (r_t - E_t \Delta p_{t+1})
\]

where the \( x_t \) variable in loglinear terms can be expressed as:

\[
x_t = \xi c_t + (1 - \xi) (1 + \phi) n_t - \frac{\lambda \xi}{\gamma_c} I_t
\]

where \( \xi = \frac{1}{1 - \phi \kappa} \). Notice that since \( \phi \) depends on the fraction of the rule of thumb consumers, so does \( \xi \).

Non Competitive Labor Market. In this appendix we interpret equation (9.21) as a log-linear approximation to a generalized wage schedule described in the main text. To obtain an expression for the aggregate Euler equation we proceed as follows. First, under the previous assumption, a log linear approximation to expression (9.2) yields

\[
E_t \Delta \tilde{c}^o_{t+1} = \frac{1}{\sigma}(r_t - E_t \Delta p_{t+1}) + x(1 - \frac{1}{\sigma})E_t \Delta n_{t+1} \tag{9.34}
\]

We log-linearize (2.2) which leads to

\[
\tilde{c}_t = \tilde{\omega}_t - \frac{\bar{r}}{1 - \bar{r}} r_t + n_t + \frac{1}{\gamma_c} \tilde{I}_t
\]

Using expression (9.21) into the previous expression yields

\[
\tilde{c}_t = \tilde{\omega}_t + (1 + \varphi) n_t + \frac{1}{\gamma_c} \tilde{I}_t \tag{9.35}
\]

Log -linearizing (9.7) yields\(^\text{27}\)

\[
\tilde{c}_t \equiv \lambda \tilde{c}_t + (1 - \lambda) \tilde{c}_t \tag{9.36}
\]

Combining expressions (9.34), (9.35), and (9.36) yields the expression for the aggregate Euler equation of the main text, i.e. (5.3).

\(^{27}\)To simplify the algebra we assume that, at the steady state, \( C = C^o = C^o \) (see also Gali, Lopez-Salido and Valles (2006)).
The response to monetary policy shocks

Impulse responses to a normalized monetary policy shock at time one. Alternative estimated models. Horizontal axes represent the time horizon after the shock measured in quarters.
The response to Technology Shocks

Impulse responses to a normalized neutral technology shock at time one. Alternative estimated models. Horizontal axes represent the time horizon after the shock measured in quarters.
Subsample Stability: All Parameter Estimates

In this small section of the Appendix we present the detailed estimates of the main parameters of the four estimated models across both subsamples.

Table 5 (Appendix). Posterior Distribution. Subsample Stability

<table>
<thead>
<tr>
<th></th>
<th>Heterogeneous Agent</th>
<th>Imperfect Labor Market</th>
<th>GHH Preferences</th>
<th>Separable Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\sigma}$</td>
<td>3.31 (2.94 - 3.74)</td>
<td>2.07 (1.79 - 2.34)</td>
<td>2.01 (1.76 - 2.13)</td>
<td>1.0 (0.16 - 0.29)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.04 (0.03 - 0.06)</td>
<td>0.13 (0.11 - 0.16)</td>
<td>0.14 (0.12 - 0.17)</td>
<td>0.04 (0.02 - 0.06)</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.46 (0.38 - 0.53)</td>
<td>0.84 (0.81 - 0.87)</td>
<td>0.81 (0.78 - 0.81)</td>
<td>0.58 (0.50 - 0.66)</td>
</tr>
<tr>
<td>$\phi_b$</td>
<td>0.11 (0.09 - 0.14)</td>
<td>0.10 (0.07 - 0.13)</td>
<td>0.06 (0.04 - 0.07)</td>
<td>0.08 (0.06 - 0.10)</td>
</tr>
<tr>
<td>$\gamma_{\pi}$</td>
<td>1.47 (1.37 - 1.58)</td>
<td>1.21 (1.16 - 1.26)</td>
<td>1.31 (1.29 - 1.33)</td>
<td>1.46 (1.41 - 1.42)</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.65 (0.60 - 0.71)</td>
<td>0.44 (0.36 - 0.52)</td>
<td>0.56 (0.53 - 0.60)</td>
<td>0.77 (0.73 - 0.81)</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>0.59 (0.50 - 0.67)</td>
<td>0.69 (0.64 - 0.73)</td>
<td>0.68 (0.66 - 0.71)</td>
<td>0.75 (0.70 - 0.80)</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>0.76 (0.70 - 0.83)</td>
<td>0.87 (0.84 - 0.90)</td>
<td>0.67 (0.61 - 0.71)</td>
<td>0.61 (0.56 - 0.66)</td>
</tr>
<tr>
<td>Log L</td>
<td>2473.9</td>
<td>2546.8</td>
<td>2477.1</td>
<td>2543.5</td>
</tr>
</tbody>
</table>

Log L values are rounded to one decimal place.