Nontradable Goods and The Real Exchange Rate*

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Abstract

How important are nontradable goods and distribution costs to explain real exchange rate dynamics? We answer this question by estimating a general equilibrium model with intermediate and final tradable and nontradable goods. We obtain reasonable estimated parameter values using Bayesian methods, and find that the estimated model can match real exchange rate persistence and, to less extent, volatility. Nontradable sector technology shocks explain about one third of real exchange rate volatility. We present impulse responses to better understand the transmission mechanism of shocks in the open economy. We also show that in order to explain the low correlation between the ratio of relative consumption and the real exchange rates across countries, fiscal policy shocks are necessary in the model, in addition to technology shocks.

Keywords: Real Exchange Rates, Non Tradable Goods, Nominal Rigidities, Bayesian Estimation.

JEL Classification: F31, F32, F41, C11.

1 Introduction

A main challenge in the empirical international macroeconomics literature is the so called "real exchange rate disconnect": models with optimizing agents have difficulty in accounting for the behavior of the real exchange rate. A related problem is that these models are not able to explain key correlations between the real exchange rate and other macroeconomic variables, nor are they able to capture the comovement between key macroeconomic variables across countries. In this paper, we investigate which modeling assumptions are more likely to help us explain the behavior of the real exchange rate between the United States and the Euro Area. In particular,

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we focus on the role of nontradable goods and distribution services.[1] After estimating the model and obtaining reasonable parameter values, we proceed to answer key questions in the literature: what drives real exchange rate fluctuations? How do productivity and fiscal shocks transmit internationally, and how do they affect, relative consumptions, the real exchange rate and the trade balance?

Our starting point to answer these questions is a two-country (USA-euro area), two-sector (tradable-nontradable) dynamic stochastic general equilibrium (DSGE) model with nominal rigidities, of the class that is now becoming mainstream in academic circles and policy institutions for macroeconomic analysis. The model includes nontradable goods because, since the work of Stockman and Tesar (1995), several authors, including Betts and Kehoe (2006) and Burstein, Eichenbaum and Rebelo (2005), have highlighted the importance in the fluctuation of the relative price of nontradable goods across countries to account for real exchange rate fluctuations. These papers find that between one third and one half of real exchange variance is explained by fluctuations in the relative price of nontradable goods to tradable goods.

Since our analysis is mainly empirical and model-based, we estimate two versions of this model: in the first one, the two sectors (tradable and nontradable) produce final consumption goods. In the second one, we introduce a nontradable intermediate input that is incorporated in the production of the final tradable good. In this second case, the idea is to capture the role of distribution costs in helping explain features of international macroeconomics, as suggested by Corsetti, Dedola and Leduc (2007) and Dotsey and Duarte (2007). Our methodology consists in estimating each model using a Bayesian approach and eleven macroeconomic series, including both the PPI and the CPI for the United States and the Euro Area. PPI inflation allows us to capture inflation in the tradable goods sector of the economy, and unlike the "goods" component of the CPI, should not include distribution costs.

Our results can be summarized as follows. First, the model parameter estimates are quite similar to what has been estimated or calibrated in the vast existing literature. Therefore, our likelihood-based method does not rely on implausible parameter values for structural coefficients such as the degree of nominal rigidity, the degree of backward looking behavior in inflation or consumption, the monetary policy rules in both countries, and the size and persistence of economic

[1] There is now a growing literature on this issue both empirical and theoretical. Empirical papers that have estimated fully specified general equilibrium international macroeconomic models include Rabanal and Tuesta (2006), Lubik and Schorfheide (2005), Adolfson et al. (2007), Justiniano and Preston (2006), Cristadoro et al. (2007), and de Walque, Smets and Wouters (2006). On the theoretical side, important recent contributions that have tried to improve international macro models include Benigno and Thoenissen (2006), Dotsey and Duarte (2007) and Corsetti, Dedola, and Leduc (2007).

shocks to explain the data. Second, we find that the version of the model without distribution costs performs better than the version with distribution costs. Since the model already includes several nominal and real rigidities, the addition of distribution costs does not help in explaining the data better, and, in fact, model fit is worse in some dimensions, including real exchange rate persistence. Third, our variance decomposition exercise (using the preferred model) shows that the nontradable sector in the model does indeed help to explain real exchange rate fluctuations: nontradable sector technology shocks explain as much as 30 percent of the fluctuation of the bilateral real exchange rate, while tradable sector technology shocks and monetary policy shocks together explain less than 2 percent. Fiscal policy shocks explain a great amount of real exchange rate fluctuations (45 percent).

Finally, our estimated model allows us to draw important implications for the behavior of the real exchange rate, the terms of trade and the trade balance. The relative price of domestic tradables decreases under a tradable sector technology shock, which is consistent with the traditional Balassa-Samuelson effect. With a productivity improvement in either the tradable and nontradable sectors, relative output, consumption and net exports increase. Finally, following a productivity shock in either sector, domestic prices decrease, and consequently the real exchange rate depreciates. This latter finding goes in line with the argument of Obstfeld and Rogoff (2004), who argue that a correction of the US external balance would entail a depreciation of the real exchange rate and worsening of the terms of trade. Yet, it goes in stark contradiction with the results reported by Corsetti, Dedola and Leduc (CDL, 2006). These authors find that the tradable sector productivity shocks are able to explain the apparent lack of risk sharing across countries (negative correlation between relative consumptions and the real exchange rate). Instead, our estimated model generates, conditional on a tradable sector productivity shock, a real exchange rate depreciation and an increase in the ratio of relative consumptions. Therefore, it is fiscal shocks that help explain the negative correlation between the real exchange rate and relative consumptions observed in the data.

The rest of the paper is organized as follows. In section 2, we present the model that we estimate. In section 3 we discuss the data, and the prior and posterior distribution of the model's parameters. In sections 4 and 5 we discuss the implications of the estimated model for real exchange rate behavior and the transmission mechanism in open economies. In section 6 we discuss the estimation of a model that incorporates distribution services, while section 7 concludes.

2 The Model

In this section, we present the model that we use for analyzing real exchange rate dynamics and the international transmission of shocks. The model is a fairly standard international macro two-country, two-sector (tradable and nontradable) economy, in the spirit of Stockman and Tesar (1995) and Benigno and Thoenissen (2006). The model includes sticky prices in both sectors, and it assumes that monetary policy is conducted with an interest rate rule of the Taylor type. Based on the arguments by Benigno and Thoenissen (2006) and on the empirical results of Rabanal and Tuesta (2006), we only explore the possibility that there are incomplete markets at the international level. Finally, we assume that the law of one price holds and intermediate firms set prices in their own currency.[2]

Since our contribution is to estimate this model using Bayesian methods and eleven observable variables, in this section we briefly present its main assumptions, parameters and functional forms, and refer the reader to the Appendix for a full-blown version of the model. In the last section of the paper, we study the effects of introducing a distribution sector in the model. We do so by following Dotsey and Duarte (2007) and assume that the production function of final tradable goods includes a portion of nontradable inputs. Finally, to keep the exposition of the model at its minimum, we only present the equations for households and firms in the home country. The expressions for the foreign country are analogous, and obtaining them is straightforward, with the appropriate change of notation.[3]

Households Representative households in the home country are assumed to maximize the following utility function:

$$U_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \psi_t \left[\log\left(C_t - b\bar{C}_{t-1}\right) - \frac{L_t^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{1}$$

subject to the following budget constraint:

$$\frac{B_t^H}{P_t R_t} + \frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t \bar{B}_t^F}{P_t Y_t}\right)} \leq \frac{B_{t-1}^H}{P_t} + \frac{S_t B_{t-1}^F}{P_t} + \frac{W_t}{P_t} L_t - C_t + \Pi_t \tag{2}$$

[2] Dotsey and Duarte (2007) show that alternative assumptions regarding pricing decisions of firms, namely producer currency pricing (PCP) and local currency pricing (LCP), are not so different for the real exchange rate dynamics. Rabanal and Tuesta (2006) find empirical evidence, estimating a DSGE model with Bayesian methods, that a model with PCP helps fitting the data better than a model with LCP when looking at euro-dollar real exchange rate dynamics.

[3] The convention will be to use an asterisk to denote the counterpart in the foreign country of a variable in the home country (i.e. if aggregate consumption is C in the home country, it will be C^* in the foreign country and so on. The same applies to the model's parameters. When there is potential for confusion we explictly clarify so.

E_0 denotes the conditional expectation on information available at date $t = 0$, β is the intertemporal discount factor, with $0 < \beta < 1$. C_t denotes the level of consumption in period t, L_t denotes labor supply. The utility function displays external habit formation with respect to the habit stock, which is last period's aggregate consumption of the economy $\bar{C}_{t-1}$. $b \in [0, 1]$ denotes the importance of the habit stock. $\varphi > 0$ is inverse elasticity of labor supply with respect to the real wage. ψ_t is a preference shock that follows an $AR(1)$ process in logs

$$\log \psi_t = \rho_\psi \log \psi_{t-1} + \varepsilon_t^\psi \tag{3}$$

In the budget constraint, W_t is the nominal wage, P_t is the consumer price index, and Π_t are real profits for the home consumer. For modelling simplicity, we choose to model incomplete markets at the international level with two risk-free one-period nominal bonds denominated in domestic and foreign currency, and a cost of bond holdings is introduced to achieve stationarity. B_t^H is the holding of the risk free domestic nominal bond and B_t^F is the holding of the foreign risk-free nominal bond expressed in units of foreign country currency. S_t is the nominal exchange rate, expressed in units of home country currency per unit of foreign country currency. R_t and R_t^* are the nominal interest rates in the home and foreign countries. The function $\Phi(.)$ depends on the net liability position (i.e. the negative net foreign asset position) of the home country, $\bar{B}_t^F$, in percent of GDP in the entire economy, and is taken as given by the domestic household. $\Phi(.)$ is a convex function that introduces the cost of undertaking positions in the international asset market, and allows to have a well-defined steady-state. In addition, it is assumed that $\Phi(0) = 1$ and that $\Phi(.)$ is a decreasing function in the neighborhood of zero. Also, while we do not make it explicit in the budget constraint (2), we assume that there are complete markets at the domestic level, such that the consumption/savings decision is the same among households in a country, and the stochastic discount factor to value future profits is also the same among households in a country.

The aggregate consumption index (C_t) is a composite of final tradable (C_t^T) and final non-tradable (C_t^N) consumption goods. We define the consumption index as

$$C_t \equiv \left[\gamma_c^{1/\varepsilon} \left(C_t^T\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma_c)^{1/\varepsilon} \left(C_t^N\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where ε is elasticity of substitution between the final tradable (C_t^T) and final non-tradable (C_t^N) goods, and γ_c is the share of final tradable goods in the consumption basket at home. In this context, the consumer price index that corresponds to the previous specification is given by

$$P_t \equiv \left[\gamma_c \left(P_t^T\right)^{1-\varepsilon} + (1-\gamma_c) \left(P_t^N\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{5}$$

where all prices are for goods sold in the home country, in home currency and at the consumer level, for both tradable and nontradable goods.

Demands for the final tradable and nontradable goods are given by:

$$C_t^T = \gamma_c \left(\frac{P_t^T}{P_t} \right)^{-\varepsilon} C_t, \text{ and } C_t^N = (1-\gamma_c) \left(\frac{P_t^N}{P_t} \right)^{-\varepsilon} C_t. \tag{6}$$

while consumption/savings decisions in home and foreign bonds are standard:

$$\lambda_t = \beta E_t \left\{ R_t \frac{P_t}{P_{t+1}} \lambda_{t+1} \right\} \tag{7}$$

$$\lambda_t = \Phi \left(\frac{S_t \bar{B}_t^F}{P_t Y_t} \right) \beta E_t \left\{ R_t^* \frac{Q_{t+1}}{Q_t} \lambda_{t+1} \right\} \tag{8}$$

where $\lambda_t = \frac{\psi_t}{C_t - b\bar{C}_{t-1}}$ is the marginal utility of consumption, and $Q_t = \frac{S_t P_t^*}{P_t}$ is the real exchange rate. Labor supply is:

$$\lambda_t \frac{W_t}{P_t} = L_t^{\varphi} \tag{9}$$

Firms There are three sectors in each country: (i) a final goods producer sector, that produces final tradable and nontradable goods for consumption by domestic households, (ii) an intermediate tradable goods sector, that produces goods that can be traded internationally to final tradable goods producers either at home or abroad, and (iii) an intermediate nontradable goods sector, that sells its production to final nontradable goods producers. We assume that the final goods producers operate under flexible prices and perfect competition, while intermediate goods producers operate under sticky prices à la Calvo with partial indexation, and monopolistic competition.

Final Goods Producers The final tradable good is consumed by domestic households. This good is produced by a continuum of firms, each producing the same variety, labelled by Y_t^T, using intermediate home $\left(X_t^h\right)$ and foreign $\left(X_t^f\right)$ goods with the following technology:

$$Y_t^T = \left\{ \gamma_x^{1/\theta} \left(X_t^h \right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta} \left(X_t^f \right)^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution between home-produced and foreign-produced imported intermediate goods. X_t^h and X_t^f denote the amount of home and foreign intermediate tradable inputs to produce the final tradable good at home, are also Dixit-Stiglitz aggregators of all types of home and foreign final goods, with elasticity of substitution σ. :

$$X_t^h \equiv \left[\int_0^1 X_t^h(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \text{ and } X_t^f \equiv \left[\int_0^1 X_t^f(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}.$$

Optimizing conditions by final tradable goods producers deliver the following demand functions:

$$X_t^h(h) = \gamma_x \left(\frac{P_t^h(h)}{P_t^h}\right)^{-\sigma} \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T, \text{ and } X_t^f(f) = (1-\gamma_x)\left(\frac{P_t^f(f)}{P_t^f}\right)^{-\sigma}\left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T$$

where

$$P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}}, \; P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}.$$

and, hence the price index of tradable goods is given by:

$$P_t^T = \left[\gamma_x \left(P_t^h\right)^{1-\theta} + (1-\gamma_x)\left(P_t^f\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

We assume that the law of one price holds for intermediate inputs, such that $P_t^h(h) = P_t^{h^*}(h)S_t$, and $P_t^f(f) = P_t^{f^*}(f)S_t$.

The production of the final nontradable good is given by:

$$Y_t^N \equiv \left[\int_0^1 X_t^N(n)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}}$$

where we assume the same elasticity $\sigma > 1$ than in the case of final tradable goods produced within the home country. The price level for nontradables is

$$P_t^N \equiv \left[\int_0^1 p_t^N(n)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$$

Intermediate Goods Producers The structure of intermediate goods producers in the two sectors is very similar. The main difference is that the intermediate non-tradable sector produces differentiated goods that are aggregated by final non-tradable good producing firms, and ultimately used for final consumption by domestic households only, while the intermediate tradable sector produces differentiated goods that can be sold to home and foreign final tradable good producers.

The production function in both sectors is linear in the labor input and has two technology shocks:

$$Y_t^N(n) = A_t Z_t^N L_t^N(n), \text{ for } n \in [0,1], \text{ and } Y_t^h(h) = A_t Z_t^h L_t^h(h), \text{ for } h \in [0,1] \tag{10}$$

where A_t is a labor augmenting aggregate world technology shock which has a unit root with drift:

$$\log A_t = g + \log A_{t-1} + \varepsilon_t^a \tag{11}$$

Hence, real variables in both countries grow at a rate g. Z_t^N and Z_t^h are country-specific, stationary productivity shocks to the nontradable and the tradable sector at time t, which evolve according to an AR(1) process in logs

$$\log Z_t^N = (1-\rho^N)\log(\bar{Z}^N)+\rho^{Z,N}\log Z_{t-1}^N+\varepsilon_t^{Z,N}, \text{ and } \log Z_t^h = (1-\rho^h)\log(\bar{Z}^h)+\rho^{Z,h}\log Z_{t-1}^h+\varepsilon_t^{Z,h} \tag{12}$$

Firms in both sectors face a Calvo lottery with partial indexation when setting their prices. In the non-tradable sector in each period, with probability $1-\alpha_N$, firms receive a stochastic signal that allows them to reset prices optimally. We assume that there is partial indexation with a coefficient φ_N to last period's sectorial inflation rate for those firms that do not get to reset prices optimally. As a result, firms maximize the following profits function:

$$Max_{P_t^N(n)} E_t \sum_{k=0}^{\infty} \alpha_N^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^N(n)\left(\frac{P_{t+k-1}^N}{P_{t-1}^N}\right)^{\varphi_N}}{P_{t+k}} - MC_{t+k}^N \right] Y_{t+k}^{N,d}(n) \right\} \tag{13}$$

subject to

$$Y_{t+k}^{N,d}(n) = \left[\left(\frac{P_t^N(n)}{P_{t+k}^N}\right) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N}\right)^{\varphi_N} \right]^{-\sigma} Y_t^N \tag{14}$$

where $Y_t^{N,d}(n)$ is total individual demand for a given type of nontradable good n, and Y_t^N is aggregate demand for nontradable goods, as defined above, and $\Lambda_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor. MC_t^N corresponds to the real marginal cost in the non-tradable sector. From cost minimization:

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t} \tag{15}$$

The evolution of the price level of nontradables is

$$P_t^N \equiv \left\{ \alpha_N \left[P_{t-1}^N \left(\Pi_{t-1}^N\right)^{\varphi_N} \right]^{1-\sigma} + (1-\alpha_N)\left(\hat{p}_t^N\right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \tag{16}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$. Similar expressions hold for the intermediate tradable sector, where the relevant parameters for price setting are α_h and φ_h, and with the appropriate change of notation in (13), (14), (15), and (16), and similar expressions hold for the foreign country.

Closing the Model The model includes government spending that is financed by lump-sum taxation. We assume that government spending is allocated between tradable and nontradable goods in the same way that private consumption is. Hence the market clearing conditions for both types of final goods, consisting of private consumption and government spending, are:

$$Y_t^T = C_t^T + G_t^T \tag{17}$$

$$Y_t^N = C_t^N + G_t^N \tag{18}$$

where G_t^N, G_t^T follow AR(1) processes in logs. For the nontradable intermediate goods, the market clearing condition is:

$$Y_t^N(n) = X_t^N(n), \text{ for all } n \in [0,1]$$

while for the intermediate tradable goods sector it is:

$$Y_t^h(h) = X_t^h(h) + X_t^{h^*}(h), \text{ for all } h \in [0,1]$$

Monetary policy is conducted with a Taylor rule that targets CPI inflation and output growth deviation from steady-state values:

$$R_t = \bar{R}^{(1-\rho_r)} R_{t-1}^{\rho_r} \left(\frac{P_t/P_{t-1}}{\Pi} \right)^{(1-\rho_r)\gamma_\pi} \left(\frac{Y_t/Y_{t-1}}{1+g} \right)^{(1-\rho_r)\gamma_y} \exp(\varepsilon_t^r) \tag{19}$$

3 Data, Priors and Posterior Distributions

We use Bayesian methods to estimate the model of the previous section. Bayesian estimation of DSGE models has now become very popular, so we leave the technical details and a discussion of its benefits aside.[4] We use the following data series for each country: per capita output growth, per capita consumption growth, CPI inflation, interest rates on 3-month T-Bills, and PPI inflation for finished industrial goods. CPI inflation is used as a measure of overall inflation, while PPI inflation tries to measure the inflation in the tradable sector of the economy. Several authors (Engel, 1999; Betts and Kehoe, 2006) have emphasized that using the "goods" component of the CPI might not be a good proxy for tradable goods because it contains distribution and retail services that are nontradable. We assume that the home country is the euro area, and the foreign country is the USA. The last series we use in the estimation procedure is the bilateral real exchange rate between the euro and the US dollar, in euros per US dollar. We multiply it by the US CPI, and divide it by the euro area CPI. Hence an increase of the real exchange rate is a depreciation. These 11 variables are our set of observable variables in the likelihood

[4] See An and Schorheide (2006), Lubik and Schorfheide (2005), and Fernández-Villaverde and Rubio-Ramírez (2004) for detailed explanations on how to implement a Bayesian approach to estimation of fully-specified dynamic stochastic general equilibrium models.

function, $\{x_t\}_{t=1}^T$. The sample period goes from 1985:02 to 2004:04. We are constrained by the availability of the PPI series for finished industrial goods in the euro area, since all other variables are available from earlier periods.

We take a linear approximation to the model's equilibrium conditions, where real variables have been normalized by the level of labor augmenting aggregate world technology process (A_t) to make them stationary. For a given set of parameters of the model Θ, we find the law of motion of the system in state-space form. Then, for a given set of observable variables of length T, $\{x_t\}_{t=1}^T$, where the dimension of x_t is less or equal to the number of shocks in the model, we can evaluate the likelihood function of the data conditional on the parameters and the model, that we denote by $L(\{x_t\}_{t=1}^T |\Theta)$, using standard Kalman filter formulae (see Ireland, 2004). Finally, by specifying prior distributions over the model's parameters, $P(\Theta)$, we can obtain the posterior distribution of the model's parameters, which is proportional to the product $L(\{x_t\}_{t=1}^T |\Theta)P(\Theta)$, applying the Metropolis-Hastings algorithm.

Before we proceed to describe the prior and posterior distribution of the model's parameters, we first discuss what parameters we calibrate. We follow Dotsey and Duarte (2007) closely, and calibrate the two economies with the same parameters. We set the share of tradable goods in the CPI to $\gamma_c = 0.44$. We set the fraction of intermediate tradable inputs in the production of final tradable goods to $\gamma_x = 0.6$. Since we are not using labor market data we calibrate the value of $\varphi = 1$, which is in line with parameter estimates obtained by Rabanal and Rubio-Ramírez (2005, 2007). We set the steady-state growth rate of the economy, g, equal to 0.5 percent, which implies that the world growth rate of per capita variables is about 2 percent per year. In order to match a real interest rate in the steady state of about 4 percent per year, we set the discount factor to $\beta = 0.995$. For reasonable parameterizations of these two variables the parameter estimates do not change significantly. Finally, the parameter χ, that measures the elasticity of the risk premium with respect to the net foreign asset position, is set equal to 0.007 based on Rabanal and Tuesta (2006).

With the previous parameters fixed in advance, Table 1 presents the prior and posterior distributions for the model's remaining parameters. In order to make the table more readable, we also include a brief description of each parameter.

Table 1: Prior and Posterior Distributions

		Prior			Posterior		
		Distr.	Mean	St.Dev	Mean	Lower	Upper
Habit formation							
b	EMU	*Beta*	0.70	0.05	0.57	0.51	0.64
b^*	USA	*Beta*	0.70	0.05	0.58	0.52	0.64
Elasticities of substitution							
θ	Home and foreign tradable int. goods	*Normal*	1.50	0.25	0.85	0.85	0.85
ε	Tradable and nontradable final goods	*Gamma*	1.00	0.25	0.13	0.09	0.18
Calvo parameters							
α_h	Tradable int. goods EMU	*Beta*	0.50	0.20	0.73	0.66	0.80
α_{f^*}	Tradable int. goods USA	*Beta*	0.50	0.20	0.48	0.38	0.59
α_N	Nontradable int. goods EMU	*Beta*	0.50	0.20	0.10	0.03	0.17
α_{N^*}	Nontradable int. goods USA	*Beta*	0.50	0.20	0.40	0.34	0.46
Indexation parameters							
φ_h	Tradable int. goods EMU	*Beta*	0.50	0.20	0.30	0.07	0.51
φ_{f^*}	Tradable int. goods USA	*Beta*	0.50	0.20	0.21	0.01	0.41
φ_N	Nontradable int. goods EMU	*Beta*	0.50	0.20	0.39	0.08	0.69
φ_{N^*}	Nontradable int. goods USA	*Beta*	0.50	0.20	0.06	0.01	0.10
Taylor rule coefficients							
ρ_r	Interest rate smoothing EMU	*Uniform*	0.50	0.29	0.76	0.71	0.82
ρ_{r^*}	Interest rate smoothing USA	*Uniform*	0.50	0.29	0.88	0.85	0.90
γ_π	Response to inflation, EMU	*Normal*	1.50	0.25	2.72	2.46	2.98
γ_{π^*}	Response to inflation, USA	*Normal*	1.50	0.25	2.05	1.73	2.34
γ_y	Response to growth, EMU	*Normal*	1.00	0.25	0.56	0.40	0.71
γ_{y^*}	Response to growth, USA	*Normal*	1.00	0.25	0.91	0.68	1.15

Table 1 (concluded): Prior and Posterior Distributions of Shocks

		Prior			Posterior		
		Distr.	Mean	St.Dev	Mean	Lower	Upper
AR Coefficients							
	Preference						
ρ_{ψ}	EMU	*Beta*	0.75	0.10	0.87	0.82	0.92
ρ_{ψ^*}	USA	*Beta*	0.75	0.10	0.88	0.83	0.93
	Technology						
$\rho^{Z,h}$	Tradable int. sector EMU	*Beta*	0.75	0.10	0.94	0.89	0.98
ρ^{Z,f^*}	Tradable int. sector USA	*Beta*	0.75	0.10	0.93	0.88	0.98
$\rho^{Z,N}$	Nontradable int. sector EMU	*Beta*	0.75	0.10	0.97	0.95	0.99
ρ^{Z,N^*}	Nontradable int. sector USA	*Beta*	0.75	0.10	0.93	0.90	0.97
	Government Spending						
$\rho^{G,T}$	Tradable sector EMU	*Beta*	0.75	0.10	0.84	0.75	0.94
ρ^{G,T^*}	Tradable sector USA	*Beta*	0.75	0.10	0.73	0.56	0.89
$\rho^{G,N}$	Nontradable sector EMU	*Beta*	0.75	0.10	0.85	0.76	0.94
ρ^{G,N^*}	Nontradable sector USA	*Beta*	0.75	0.10	0.93	0.89	0.97
Standard Deviations of Shocks (in percent)							
	Preference						
ε_t^{ψ}	EMU	*Gamma*	1.00	0.50	1.89	1.43	2.36
$\varepsilon_t^{\psi^*}$	USA	*Gamma*	1.00	0.50	1.91	1.50	2.30
	Technology						
$\varepsilon^{Z,h}$	Tradable int. sector EMU	*Gamma*	0.70	0.30	1.38	1.06	1.71
ε^{Z,f^*}	Tradable int. sector USA	*Gamma*	0.70	0.30	1.72	1.37	2.05
$\varepsilon^{Z,N}$	Nontradable int. sector EMU	*Gamma*	0.70	0.30	1.81	1.54	2.08
ε^{Z,N^*}	Nontradable int. sector USA	*Gamma*	0.70	0.30	1.02	0.82	1.22
ε^{a}	Permanent technology shock.	*Gamma*	0.70	0.30	0.36	0.19	0.53
	Government Spending						
$\varepsilon^{G,T}$	Tradable sector EMU	*Gamma*	1.00	0.50	1.00	0.45	1.65
ε^{G,T^*}	Tradable sector USA	*Gamma*	1.00	0.50	0.69	0.15	1.22
$\varepsilon^{G,N}$	Nontradable sector EMU	*Gamma*	1.00	0.50	3.05	2.68	3.46
ε^{G,N^*}	Nontradable sector USA	*Gamma*	1.00	0.50	4.07	3.57	4.60
	Monetary policy						
ε^{r}	EMU	*Gamma*	0.40	0.20	0.16	0.12	0.19
ε^{r^*}	USA	*Gamma*	0.40	0.20	0.11	0.09	0.13

Since we are mostly interested in understanding the implications of the model for real exchange rate dynamics and the international transmission of shocks, we briefly comment on the parameter estimates. Overall, they are quite similar to what has been obtained in the literature that estimates open economy models with Bayesian methods.[5] The estimates for the degree of habit

[5] See Lubik and Schorfheide (2005), Rabanal and Tuesta (2006) and Cristadoro et al. (2007).

formation are quite similar in both countries, of 0.57 in the United States and of 0.58 in the euro area, respectively. The elasticity of substitution between home and foreign tradable inputs, θ, is estimated at 0.85, a value much smaller than the prior mean of 1.5, which was chosen according to Chari, Kehoe, and McGrattan (2002). However, this value is higher than that obtained by Rabanal and Tuesta (2006) and Lubik and Schorfheide (2005) in a model with tradable goods only. As it will become clearer later, the higher estimated value for this elasticity stems from endogenous volatility that nontradable goods adds to the model, hence making less necessary a small value of θ to account for real exchange volatility. On the other hand, the elasticity of substitution between tradable and nontradable final consumption goods, ε, is estimated to be quite low, with a posterior mean of 0.13, which is much lower than the prior mean, of 1, and the value typically used calibrated exercises in the literature of 0.44, following Stockman and Tesar (1995). The estimated Phillips Curves suggest that prices in the USA are reset optimally about every 2 quarters in both sectors, with a low degree of backward looking indexation $\left(\varphi_{N^*}, \varphi_{f^*}\right)$, between 0.06 in the nontradable sector and 0.21 in the tradable sector. The Phillips Curves in the euro area are more heterogeneous: the estimated probability of not resetting prices is 0.73 in the tradable sector, while we obtain a surprisingly low coefficient for the nontradable sector, where the posterior mean is 0.1, much lower than the prior mean of 0.5. Backward looking behavior is higher than in the case of the USA, with coefficients of 0.3 in the tradable sector and 0.4 in the nontradable sector. Finally, the coefficients on the Taylor rule are quite similar to previous estimates in the literature for the sample period that we use, starting in 1985, with coefficients on the response of nominal interest rates to inflation of 2 in the United States (Clarida, Galí and Gertler, 2000) and even higher in the euro area. Regarding the exogenous processes, all shocks are estimated with high, but reasonable, persistence. The technology shock in the intermediate nontradable sector has the highest persistence, with a posterior mean of 0.97, while the persistence of all the other shocks ranges between that value and 0.73 for the tradable sector fiscal shock in the USA. The high persistence in preference and technology shocks in the nontradable sector in US might explain why is the backward behavior in price setting to be unimportant here. Similar results have been found by Ireland (2006) for an estimated closed economy using US data.

4 Implications for Real Exchange Rate Dynamics: Second Moments and Variance Decomposition

After taking a linear approximation to the steady-state conditions, the equations determining the real exchange rate are as follows. First, combining the consumption Euler equations for both households, we obtain that:

$$E_t\left(q_{t+1}-q_t\right) = \left[\frac{(1+g)\,E_t\Delta c_{t+1}-b\Delta c_t}{(1+g-b)}\right]-\left[\frac{(1+g)\,E_t\Delta c^*_{t+1}-b^*\Delta c^*_t}{(1+g-b^*)}\right] + \left(1-\rho_\psi\right)\widehat{\psi}_t-\left(1-\rho^*_\psi\right)\widehat{\psi}^*_t+\chi b_t \tag{20}$$

where q_t is the real exchange rate, c_t and c_t^* are consumption in the euro area and in the United States, $b_t=\left(\frac{S_tB_t^F}{P_t}\right)Y^{-1}$ is the net foreign asset position as percent of GDP, where $\chi\equiv-\Phi'(0)\,Y$, and $\widehat{\psi}_t$ and $\widehat{\psi}^*_t$ are the preference shocks (all expressed in log deviations from steady-state values).[6] Therefore, in principle, if consumption growth in both areas is not related to the real exchange rate, the preference shocks should allow us to explain the data in case of misspecification. In addition, by taking the definition of the real exchange rate as the ratio of price levels expressed in common currency, and by using the definition of the CPIs in both countries and the definitions of the price level of tradable goods, we obtain the following expression:

$$q_t=(2\gamma_x-1)t_t+(1-\gamma_c)[(t_t^T-t_t^N)-(t_t^{T^*}-t_t^{N^*})] \tag{21}$$

where t_t is the terms of trade, defined as the price of imports minus the price of exports, $t_t^i=p_t^i-p_t$, $i=T,N$ is the relative price of tradables and nontradables in the CPI in the euro area, and similarly $t_t^{i^*}=p_t^{i^*}-p_t^*$, $i=T,N$ is the relative price of tradables and nontradables in the CPI in the United States. Therefore, the shocks that have the potential to drive the terms of trade, or that move prices of tradable and nontradable goods in both countries in different directions, are also likely to affect the behavior of the real exchange rate. Indeed, the presence of nontradable goods helps in breaking the strong correlation between the real exchange rate and the terms of trade implied by a model without nontradable goods: in that case, $\gamma_c=1$, and $q_t=(2\gamma_x-1)t_t$. Furthermore, as pointed out by Dotsey and Duarte (2007), the presence of nontradable goods lowers the correlation of real variables with international relative prices, helping the model to better explain the data. In the next sub-section we analyze some second moments and evaluate how well the model works in the previous mentioned dimensions.

In the Bayesian approach, goodness of fit and model comparisons are performed using the marginal likelihood, which updates the researcher's prior beliefs on which model is closer to the true one after observing the data. Fernández-Villaverde and Rubio-Ramírez (2004) show that, in the Bayesian framework, model comparison is consistent when models are misspecified, which is

[6] The evolution of net foreign assets over GDP is: $\beta\widetilde{b}_t=\frac{1}{1+g}\widetilde{b}_{t-1}+\frac{X^f}{Y}\left(\widetilde{x}_t^{h^*}-\widetilde{x}_t^f-t_t\right)$ where $\frac{X^f}{Y}$ is the imports-GDP ratio, $\widetilde{x}_t^{h^*}$ is exports of intermediate tradable goods, $\widetilde{x}_t^f$ is imports, and t_t is the terms of trade. Appendix B details the full set of loglinearized conditions of the model.

typically the case. However, the marginal likelihood, which averages all possible likelihood values implied by the model across the parameter space, using the prior as a weight, is a summary statistic of overall goodness of fit. In this section, we focus instead on a subset of second moments that are key in the international macroeconomics literature. In Table 2 we present some selected posterior second moments of the raw data, while in Table 3 we report selected posterior second moments of HP-filtered real variables.

Table 2: Second Moments in the Model and in the Data

	Euro Area					United States					
Std.Dev. (in %)	Y	C	R	CPI	PPI	Y	C	R	CPI	PPI	Q
Data	0.51	0.51	0.77	0.27	0.33	0.50	0.48	0.49	0.36	0.79	4.64
Model	0.74	0.87	0.57	0.55	0.60	0.59	0.75	0.49	0.84	1.23	3.34
Variance Decomp.											
Preferences	9.6	11.0	68.3	25.3	19.8	4.1	13.2	65.9	43.5	14.4	23.6
Tech. Tradable	7.4	8.4	4.3	4.1	52.9	11.9	14.0	5.0	12.1	63.8	0.9
Tech. Nontradable	49.5	56.5	19.2	33.4	18.3	20.4	55.9	22.7	23.7	13.1	31.8
Fiscal Policy	26.4	19.5	7.1	16.8	5.5	51.8	10.4	5.8	4.7	2.9	43.5
Monetary Policy	0.2	0.2	0.3	16.2	3.1	0.8	1.2	0.5	14.3	5.1	0.2
Unit Root Shock	7.0	4.4	0.9	4.3	0.3	11.0	5.4	0.1	1.6	0.8	0.1

Note: Y is output, C is consumption, R is nominal interest rate. Q is the real exchange rate. Moments for R are based on the level of this variable, in all other cases they are based on their quarterly growth rate.

Regarding the variables as they are entered in the estimation procedure, the model overpredicts the volatility of consumption growth, output out and CPI and PPI inflation in both countries, while it underpredicts the volatility of nominal interest rates and the real exchange rate (Table 2). The model has some trouble in capturing the "great moderation" in inflation rates, specially in the euro area. Yet, using a longer period that includes the 1970s makes the model fit the inflation data better as documented by Rabanal and Tuesta (2006). To explain which shocks drive the behavior of macroeconomic variables, we perform a variance decomposition exercise and then add up shocks across countries.[7] Most important for the purpose of this paper, we examine what is the role of each shock in explaining real exchange rate fluctuations. In this case, fiscal policy shocks (mostly in the tradable sector) explain 43.5 percent of the variance of the real exchange rate, while technology shocks in the nontradable sector explain 31.8 percent, and preference shocks explain 23.6 percent. The other shocks (monetary policy, innovations to the permanent technology shock, and the tradable sector technology shock) explain about the re-

[7] That is, the contribution of the "Preference" shock adds up the contribution of the EMU preference shock and the USA preference shock. The only exception is the "Fiscal Shock" for which we have aggregated across countries and sectors.

maining 1 percent. These results confirm the findings of Rabanal and Tuesta (2006) with a model with tradable goods only. Of course, in that case we were not able to tell what sector the shocks belonged to, but we assigned an important contribution (about 40 percent each) to technology and demand factors. These results are also in stark contrast with Cristadoro et al. (2007), who estimate a model similar to this one but with distribution costs and uncovered interest rate parity (UIP) shocks. Despite introducing such complexities, real exchange rate volatility is mostly explained by the UIP shock (75 percent). Back to our estimation, nontradable technology shocks, fiscal shocks and preference shocks are able to explain a large fraction of the volatility of most variables. Note also that the tradable sector technology shocks only explain an important fraction of tradable (PPI) inflation in both countries. Therefore, introducing a nontradable sector in the model does seem to help explain the data better. Also, the monetary policy shock does explain a significant fraction of CPI inflation in both countries, about 15 percent.

Table 3: Second Moments in the Model and in the Data

Correlation	Y,Y*	C,C*	C-C*,Q	Y-Y*,Q	Q,Q_{-1}
Data	0.30	0.18	0.01	0.15	0.78
Model	0.36	-0.28	0.05	0.20	0.78
Preferences	0.85	-0.70	-0.97	-0.96	0.77
Tech. Tradable	0.96	0.87	0.84	0.89	0.62
Tech. Nontradable	0.12	-0.49	0.89	0.91	0.77
Fiscal Policy	0.29	-0.55	-0.90	-0.38	0.79
Monetary Policy	0.95	0.91	0.81	0.85	0.23
Unit Root Shock	1.00	1.00	0.78	-0.61	0.18

Note: Y is output, C is consumption, Q is the real exchange rate. All moments are computed by simulating the model 1000 times with 85 periods at the posterior mode and applying the HP filter.

The model does a good job in explaining the international transmission of shocks. It does a better job in explaining the correlation of output across countries than that of consumption. Indeed, the latter in the model it is mildly negative while in the data it is mildly positive. The model is also able to explain the so-called *consumption-real exchange rate anomaly.* In the sample period that we use, the correlation between the ratio of relative consumptions and the real exchange rate is basically zero.[8] The fact that the model can match a basically zero correlation should not mask that the transmission mechanisms underlying this result are very different. While technology shocks in both sectors, monetary policy and unit root shocks deliver a high and positive correlation between these two variables, preference and fiscal policy shocks deliver

[8] Adding the seventies and mid-eighties sample, as in Rabanal and Tuesta (2006), delivers a negative correlation of -0.17, that a model with incomplete markets and tradable goods can match.

a highly negative correlation. Therefore, any model that tries to be successful in explaining this correlation must have a combination of the two, even when the model includes nontradable goods. The same result applies when looking at the correlation between relative outputs and the real exchange rate. Finally, we would like to remark that the model is able to fit real exchange rate persistence, with a first autocorrelation in the HP-filtered real exchange rate in the model and in the data of 0.78. Also, the three shocks that explain most of real exchange rate volatility are able to explain its persistence.

5 Implications for the Transmission Mechanism

Having shown what are the three shocks that explain the behavior of the real exchange rate in the previous section, we now turn to discuss the impulse responses to a nontradable technology shock, a tradable sector demand (fiscal) shock, and a preference shock in the euro area. In Figure 1 we depict the effects of a positive (one standard deviation) nontradable sector technology shock. As a result, consumption and output increase in the euro area. The real exchange rate and the terms of trade depreciate following the shock, and the relative price of nontradables ($REL^N = \frac{P^N}{P^T}$) falls in the euro area where as it increases in the USA. From equation (21) , the RER dynamics can be decomposed in the terms-of-trade effect, $(2\gamma_x - 1)\, t_t$, and the movements of relative prices of tradable to nontradable goods in both countries. We can further rearrange (21) to get:

$$q_t = (2\gamma_x - 1)t_t + (1 - \gamma_c)(rel_t^{N^*} - rel_t^N) \qquad (22)$$

where $rel_t^N = p_t^N - p_t^T$ and $rel_t^{N^*} = p_t^{N^*} - p_t^{T^*}$. In this case, both effects move the real exchange rate in the same direction. The terms of trade depreciate because of the associated nominal exchange rate depreciation. This causes consumption to fall in the United States, and also the relative price of tradable goods to increase. Finally, there is a small improvement of the trade balance but of several orders of magnitude smaller than all in other variables. An estimated θ close to one causes the trade balance to barely move in all the exercises that we show. Note that this shock implies a positive correlation between both the real exchange rate and the terms of trade with both relative output and consumption.The impulse response to a tradable sector technology shock (not shown) displays similar behavior of the main variables, except for the relative prices of nontradable to tradable goods[9]. Our estimated impulse responses are in line

[9] For robustness, we have also performed an estimation using the terms of trade as an observable variable. Qualitatively, the impulse-responses do not change. Results are available upon request. It is worth to mention that Dotsey and Duarte (2007) find similar dynamics using a calibrated model for the US and OECD countries.

with the argument of Obstfeld and Rogoff (2004), who argue that a correction of the US external balance would entail a depreciation of the real exchange rate and worsening of the terms of trade. Instead, our empirical results challenge those of Corsetti, Dedola and Leduc (2006) which find exactly the opposite of what is postulated by Obstfeld and Rogoff (2004). Our results are more in line with those of Obstfeld Rogoff (2004) given the identification restrictions that our DSGE model imposes, while Corsetti, Dedola and Leduc (2006) consider long-run restrictions to identify permanent technology shocks.

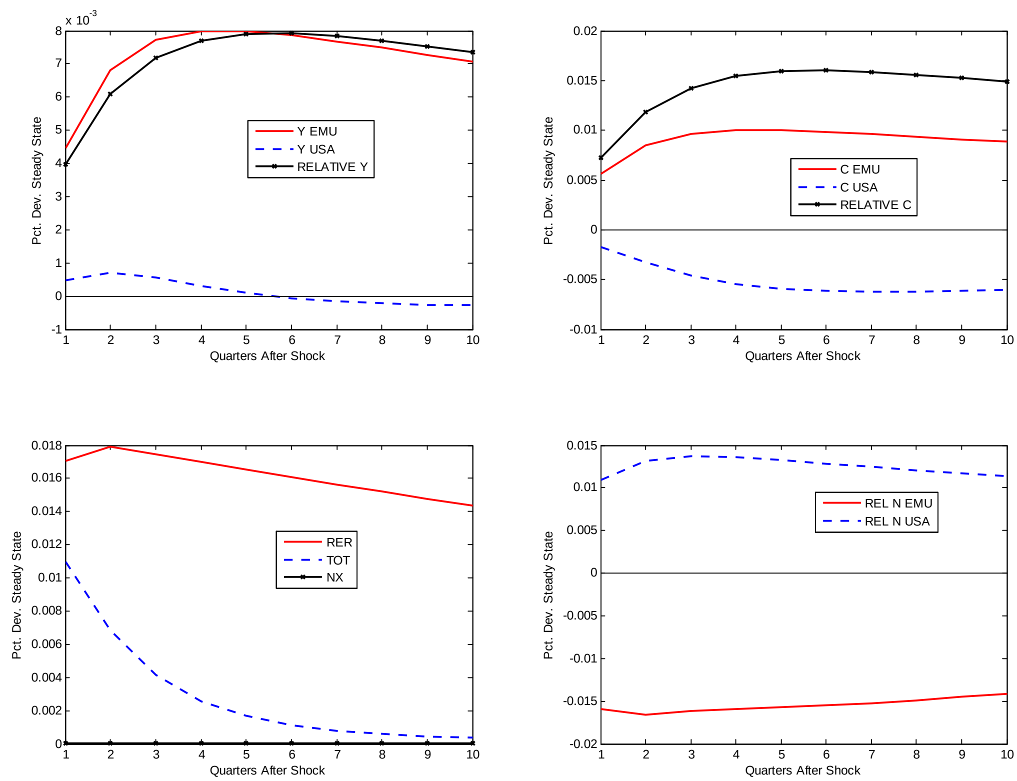

Figure 1: Impulse response to a nontradable technology shock in the euro area

In Figure 2 displays the impulse response to a fiscal shock in the tradable sector in the euro area. In this case, consumption declines in the euro area and increases in the USA, while the euro depreciates in real terms. The terms of trade also depreciates which boosts consumption in USA. Why do both the real exchange rate and the terms of trade depreciate? Since the model features, infinitely-lived Ricardian households, the positive fiscal shock induces a negative wealth effect in Euro area, therefore, agents work more and consume less today. Hence, the labor supply increases causing a reduction in real wages that further implies a reduction in marginal costs in both sectors. Thus, domestic prices (tradable and nontradable) decrease which triggers both a real exchange rate and terms of trade depreciation.

Note also that the ratio of relative consumptions decreases with the depreciation, and implies a strong negative correlation between the real exchange rate and relative consumptions across countries. Negative wealth effects cause consumption to decrease in the Euro Area more than the reduction of consumption in USA, thus relative consumption falls. Hence, as noted above, the presence of both fiscal and preference shocks are necessary to explain the RER observed dynamics. The reduction in consumption in the Euro area, is consistent with the prediction of a standard RBC model that features infinitely-lived Ricardian households that based their decisions on the intertemporal budget constraint. In particular, ceteris paribus, an increase in government spending lowers the present value of income generating a negative wealth effect that induces a cut in consumption. In our model, this effect is so strong that it implies a reduction of output as well. Finally, the trade balance deteriorates slightly being consistent with the evidence reported in Monacelli and Perotti (2006). Therefore, it is crucial to have fiscal shocks in the model, in order to be able to explain the real exchange rate-relative consumption anomaly.[10]

[10] We have estimated the model assuming non-separable preferences, along the lines of Monacelli and Perotti (2006). Under this preference specification, we were able to reproduce impulse responses conditional to both fiscal and tradable sector technology shocks that are consistent with the VAR evidence reported in Monacelli and Perotti (2006) and CDL (2006), respectively. Yet, the marginal likelihood of the model with separable preferences decreases substantially and the overall fit of this specification underperforms our benchmark model. Results are available upon request from the authors.

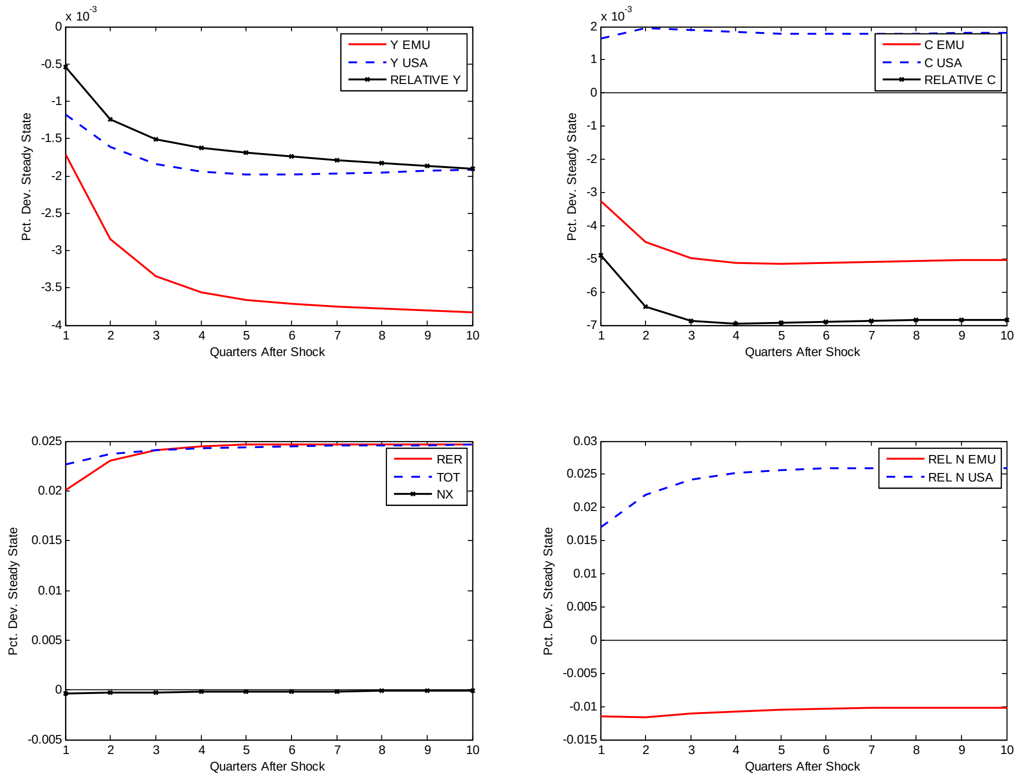

Figure 2: Impulse response to a tradable demand shock in the euro area

In Figure 3, we plot the impulse response to a preference shock, which has very similar effects to the fiscal shock regarding the implied comovement between the real exchange rate and relative consumption. However, unlike fiscal shocks it induces a positive wealth effects generating instead a real exchange rate appreciation. By increasing the marginal utility of consumption, consumption itself increases in the euro area, and the real exchange rate and terms of trade appreciate, which reduces consumption but increases output in the United States. Output in USA increases due to the positive demand shocks that implies an increase of demand of both domestic and foreign goods. This also opens a small trade deficit for the Euro area. Why both real exchange and the terms of trade appreciates?. The preference shock induces a positive wealth effect that is reflected in higher consumption. This increase of consumption leads to an increase in wages, marginal cost increases and consequently prices increase in both sectors. The price increase induces both a real exchange rate and terms of trade appreciation. Again, as noted above, we obtain a negative correlation between the real exchange rate and the ratio of relative consumptions, making this shock necessary to explain the data. At the same time, the relative price of nontradables increases in the euro area, but decreases in the United States.

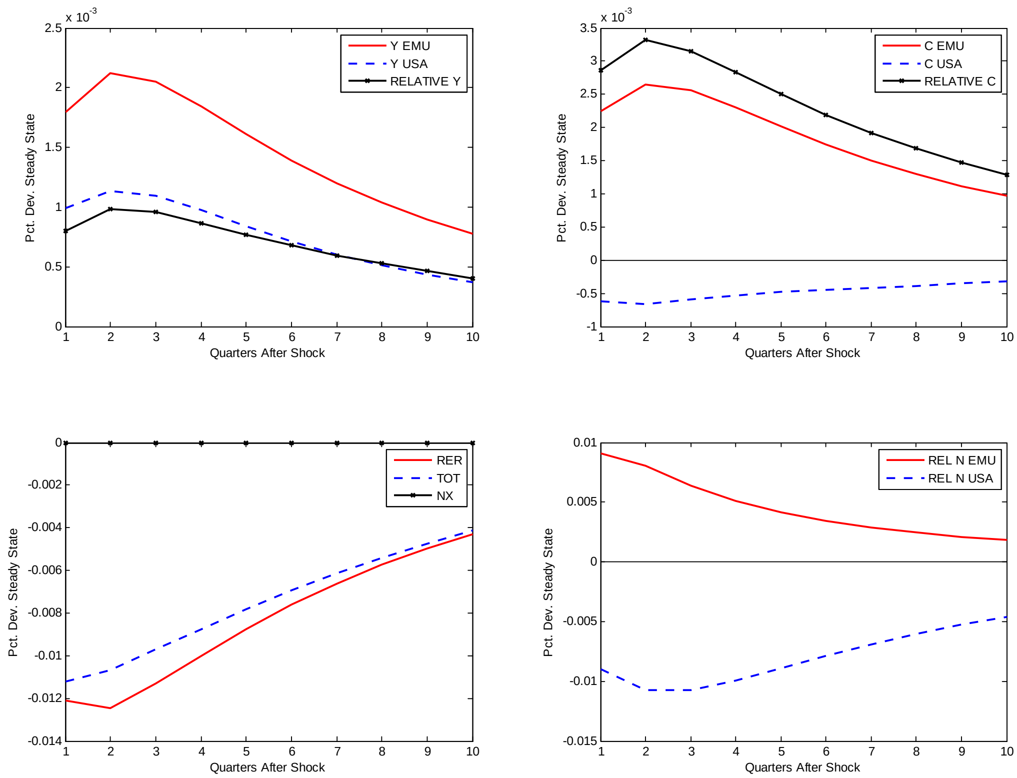

Figure 3: Impulse response to a preference shock in the euro area.

To further gauge the importance of the previous shocks in accounting for the historical RER dynamics, Figure 4 displays the observed value of the variation in the real exchange rate, together with the values with only tradable demand, nontradable technology, preference, and the other shocks, according to our estimated model.[11] This exercise allow us to identify the nature of the

[11] We use the Kalman filter to recover the sequence of shocks. We basically obtain the cyclical components of the change in the real exchange rate associated with each shock, according to our estimated model at its posterior mean.

shocks that have played a dominant role as a source of the real exchange rate dynamics.

It is clear that demand shocks explain a great fraction of the real exchange rate fluctuations being positive correlated with the real exchange rate, results that are consistent with the evidence illustrated above. Hence, the model with demand shocks provides a very good approximation to the data. But, as we mentioned before, a model with only demand shocks would imply a too negative correlation between relative consumptions and the real exchange rate, so this is why other shocks in the model are need. When the model is simulated with the nontradable component only, we can see that it is also able to capture some comovement with the actual series. On the other hand, when the model is simulated with preference shocks only, or the rest of shocks, the behavior of the change in the real exchange rate in the model and in the data is quite different.

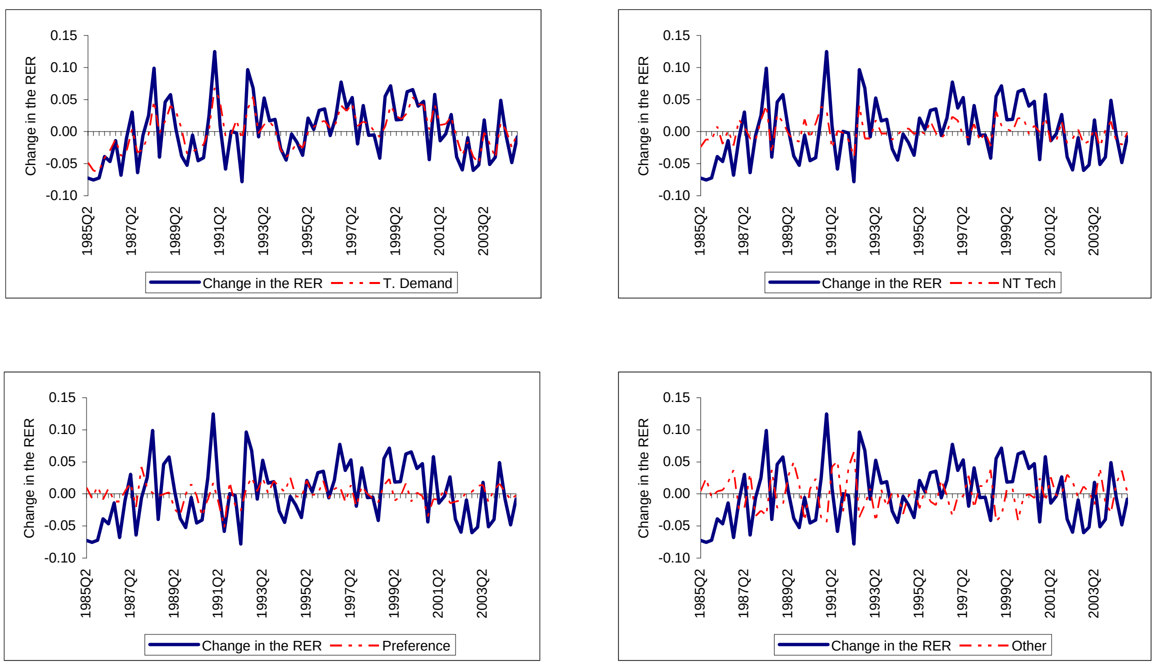

Figure 4: Decomposition of the Real Exchange Rate.

6 The Role of the Distribution Sector

In recent papers, Corsetti, Dedola and Leduc (2007) and Dotsey and Duarte (2007) have emphasized the role of the distribution sector in explaining real exchange rate dynamics. Here, we follow Dotsey and Duarte (2007) and estimate two different versions of that model. In the first one, we assume that the final tradable consumption good includes a nontradable intermediate input, and is produced under monopolistic competition (there is product differentiation). In the second case, we further assume that the final tradable good is also priced with a Calvo-type restriction.

We modify the model along the following lines. The final tradable good is consumed by domestic households. This good is produced by a continuum of firms, each producing a differentiated variety, labelled by $Y_t^T(i)$, $i \in [0,1]$. Each firm combines a composite of home and foreign intermediate tradable goods X^T, with a composite of intermediate nontradable goods X^N with the following production function:

$$Y_t^T(i) = \left\{ \gamma_y^{1/\varepsilon_Y} \left[X_t^T(i) \right]^{\frac{\varepsilon_Y - 1}{\varepsilon_Y}} + (1-\gamma_y)^{1/\varepsilon_Y} \left[X_t^N(i) \right]^{\frac{\varepsilon_Y - 1}{\varepsilon_Y}} \right\}^{\frac{\varepsilon_Y}{\varepsilon_Y - 1}}$$

where ε_y is the elasticity of substitution between tradable and nontradable intermediate goods, and γ_y is the share of tradable intermediate goods in the production function. The nontradable component can be seen as distribution services needed to bring the final consumption good to consumers. This production structure somewhat generalizes, but does not nest, Corsetti, Dedola and Leduc (2007), and implies a wedge between the price of the CES aggregate of tradable inputs and the price paid by the final consumer, due to distribution costs. When $\gamma_y = 1$, we go back to the model of section 2, but with product differentiation and monopolistic competition in the final tradable goods sector.

The local nontradable intermediate input is a Dixit-Stiglitz aggregate of all nontradable varieties, with the same elasticity than the consumption aggregate:

$$X_t^N(i) \equiv \left[\int_0^1 X_t^N(i,n)^{\frac{\sigma-1}{\sigma}} dn \right]^{\frac{\sigma}{\sigma-1}}$$

where $X_t^N(i,n)$ is the amount of intermediate nontradable input n by final good producer i. The price level P_t^N is the same as the one defined in section 2.

The composite of home and foreign intermediate tradable goods is given by:

$$X_t^T(i) = \left\{ \gamma_x^{1/\theta} \left[X_t^h(i) \right]^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta} \left[X_t^f(i) \right]^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

The definition of the composite of home and foreign intermediate goods follows from section 2.

Taking a linear approximation to the firm's optimizing conditions, when prices of the final tradable good are flexible, delivers the following inflation rate for the final tradable goods sector:

$$\triangle p_t^T = \gamma_y \left[\gamma_x \triangle p_t^h + (1-\gamma_x)\left(\triangle p_t^f + \Delta s_t\right)\right] + (1-\gamma_y)\triangle p_t^N \tag{23}$$

such that the final tradable goods sector includes a nontradable component. Further, if we assume that there are sticky prices in the final tradable good sector, inflation dynamics in the final goods tradable sector are given by

$$\triangle p_t^T - \varphi_T \triangle p_{t-1}^T = \beta E_t\left(\triangle p_{t+1}^T - \varphi_T \triangle p_t^T\right) + \kappa_T\left(mc_t^T - t_t^T\right) \tag{24}$$

where $\kappa_T = (1-\alpha_T)(1-\beta\alpha_T)/\alpha_T$, $mc_t^T = \gamma_y\left(t_t^X + t_t^T\right) + \left(1-\gamma_y\right)t_t^N$,and $t_t^X = [\gamma_x p_t^h + (1-\gamma_x)\left(p_t^f + s_t\right)] - p_t^T$.

Rather than presenting the full set of parameter estimates, that are available upon request, we compare how the models with a distribution sector fit the data, and in particular some selected moments of the data. In Table 4 we present the marginal likelihoods of the three models (baseline, distribution sector with final flexible prices, and distribution sector with final sticky prices).[12]

Table 4 : Model Comparison

	Data	Baseline	Distribution	Distr. with Sticky Prices
Marginal Likelihood	-	3292.2	3222.0	3261.4
$Std(Q/Q_{-1})$	4.64	3.34	3.77	4.34
Percent variance explained by				
Preference shocks	-	23.6	81.1	63.1
Nontradable tech. shocks	-	31.8	13.5	6.4
Fiscal shocks	-	43.5	0.8	0.6
Corr (Q, Q_{-1})	0.78	0.78	0.75	0.68
Corr $(C/C^*, Q)$	0.01	0.05	-0.14	-0.23

Note: $Std(Q/Q_{-1})$ is based on raw data, Corr (Q, Q_{-1}) and Corr $(C/C^*, Q)$ is based on HP-filtered data.

To compare overall performance, we focus on the posterior odds ratio between two models A and B:

$$\frac{\Pr(\text{model} = A|\left\{x_t\right\}_{t=1}^T)}{\Pr(\text{model} = B|\left\{x_t\right\}_{t=1}^T)} = \frac{\Pr(A)}{\Pr(B)}\frac{L(\{x_t\}_{t=1}^T|\text{model} = A)}{L(\{x_t\}_{t=1}^T|\text{model} = B)}$$

[12] The additional parameters γ_y and the fraction of intermediate goods that is used to produce the final tradable good are taken from Dotsey and Duarte (2007). Hence we calibrate γ_y to 0.62, and the fraction of nontradable production that is used as an input in the production of final traded goods to $\frac{X^N}{Y^N} = 0.4$. We also estimated versions of the two distribution cost models where we estimated those parameters. The qualitative results did not change, and model fit did not improve significantly. In addition to these two parameters, in the model with a distribution sector and sticky prices, we also estimate α_T and φ_T with the same priors than the other Calvo lotteries and backward looking parameters of Table 1. We also estimate the elasticity of substitution between tradable and nontradable inputs, ε_y.

If one does not have strong views about which model is the true one before observing the data, then $\Pr(A) = \Pr(B)$, and the researcher updates her beliefs on which model is the true one after observing the data according to the Bayes factor, which is the ratio of marginal likelihoods between two models $\frac{L(\{x_t\}_{t=1}^{T}|\text{model}=A)}{L(\{x_t\}_{t=1}^{T}|\text{model}=B)}$. Introducing a distribution sector in the model does not improve the model fit: a log Bayes factor of 70.2 (=3292.2-3222) implies that the researcher would need to have a prior probability that the distribution model is the true one about exp(70) times larger than the prior probability over the baseline model. When we introduce sticky prices in the final goods sector, model fit improves with respect to the model with flexible goods prices, but does not reach the value of the baseline model. Hence we conclude that the introduction of a distribution sector to the two-sector economy does not improve its capability of explaining the data, beyond that already included in a two-sector model with tradable and nontradable goods.

Finally, Table 4 includes some additional posterior second moments that international business cycle models would want to replicate. As we can see, the addition of a distribution sector, and afterwards sticky prices in the final goods tradable sector, increases the volatility of the real exchange rate to values that are closer to those in the data. On the other hand, as we introduce these features into the models, they are not so capable of explaining persistence. An additional unpleasant result is that, in the models with distribution costs, real exchange rate dynamics end up being explained by preference shocks, which have a more difficult interpretation than technology or demand shocks. This result could be closely linked to Cristadoro et al. (2007), who estimate a model with distribution costs à la CDL and local currency pricing, but still find that 75 percent of the volatility of the real exchange rate is due to shocks to the uncovered interest rate parity condition.

7 Concluding Remarks

In this paper we have examined the ability of models with tradable and nontradable goods to fit the data. Our main result is that we are able to match real exchange rate persistence, and to less extent, its volatility, with a medium-scale macroeconomic model estimated with Bayesian methods. At the same time, this result is based on estimated parameters that can be considered to be “reasonable” in the literature. We have found that it is mostly technology shocks in the nontradable sector, and demand shocks in the tradable sector the ones that seem to explain most of the behavior of the real exchange rate. When we have estimated versions of the model with distribution services and sticky prices in the final tradable good sector, we have not obtained a better model fit.

Estimation of DSGE models with several nominal and real rigidities tend to reveal that not all features are necessary to fit the data when priors are not too informative (see Galí and Rabanal, 2005; or Rabanal and Tuesta, 2006). On the other hand, estimated models where priors are much more informative tend to validate the rigidities in place (see Smets and Wouters, 2003; and Adolfson et al., 2007). In our case, we find that distribution services on top of several other rigidities is not necessary, but this does not mean it is not a feature of relevance in international macroeconomics, or to explain the apparent deviation from the law of one price in industry-level data (Betts and Kehoe, 2006). In any case, we have found that a two-sector two-country model in the spirit of Stockman and Tesar (1995), complemented with nominal rigidities and habit formation, seems to do a good job in explaining the data.

The presence of preference shocks and incomplete markets (or, in other papers, of uncovered interest rate parity shocks) helps us overcome the severe misspecification embedded in perfect risk sharing conditions across countries, which is at odds with the data. In addition, the "Great Moderation" that has reduced the volatility of real and nominal variables in the United States and the euro area has not affected real exchange rates. In ongoing work, Rabanal, Rubio-Ramírez and Tuesta (2007) introduce stochastic volatility in an open economy DSGE model, and estimate it.

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A Appendix: The Baseline Model

In this appendix, we present the full version of a model with tradable and nontradable final consumption goods, in the spirit of Stockman and Tesar (1995). We introduce sticky prices in both sectors to be able to study inflation dynamics and their role in affecting the real exchange rate.

A.1 Households

A.1.1 Preferences

Representative households in the home country are assumed to maximize the following utility function:

$$U_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \psi_t \left[\log\left(C_t - b\bar{C}_{t-1}\right) - \frac{L_t^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{25}$$

E_0 denotes the conditional expectation on information available at date $t = 0$, β is the intertemporal discount factor, with $0 < \beta < 1$. C_t denotes the level of consumption in period t, L_t denotes labor supply. The utility function displays external habit formation with respect to the habit stock, which is last period's aggregate consumption of the economy $\bar{C}_{t-1}$. $b \in [0, 1]$ denotes the importance of the habit stock. $\varphi > 0$ is inverse elasticity of labor supply with respect to the real wage. ψ_t is a preference shock that follows an $AR(1)$ process in logs

$$\log \psi_t = \rho_\psi \log \psi_{t-1} + \varepsilon_t^\psi \tag{26}$$

We define the consumption index as

$$C_t \equiv \left[\gamma_c^{1/\varepsilon} \left(C_t^T\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma_c)^{1/\varepsilon} \left(C_t^N\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is elasticity of substitution between the final tradable (C_t^T) and final non-tradable (C_t^N) goods, and γ_c is the share of final tradable goods in the consumption basket at home.

In this context, the consumer price index that corresponds to the previous specification is given by

$$P_t \equiv \left[\gamma_c \left(P_t^T\right)^{1-\varepsilon} + (1-\gamma_c) \left(P_t^N\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}},$$

where all prices are for goods sold in the home country, in home currency and at consumer level, for both tradable and nontradable goods.

Demands for the final tradable and nontradable goods are given by:

$$C_t^T = \gamma_c \left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t,$$
$$C_t^N = (1-\gamma_c)\left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t.$$

A.1.2 Incomplete Asset Markets

For modelling simplicity, we choose to model incomplete markets with two risk-free one-period nominal bonds denominated in domestic and foreign currency, and a cost of bond holdings is introduced to achieve stationarity[13]. Then, the budget constraint of the domestic households in real units of home currency is given by:

$$\frac{B_t^H}{P_t R_t} + \frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t \bar{B}_t^F}{P_t Y_t}\right)} \leq \frac{B_{t-1}^H}{P_t} + \frac{S_t B_{t-1}^F}{P_t} + \frac{W_t}{P_t} L_t - C_t + \Pi_t \tag{27}$$

where W_t is the nominal wage, and Π_t are real profits for the home consumer. B_t^H is the holding of the risk free domestic nominal bond and B_t^F is the holding of the foreign risk-free nominal bond expressed in foreign country currency. S_t is the nominal exchange rate, expressed in units of home country currency per unit of foreign country. The function $\Phi(.)$ depends on the net liability position (i.e. the negative net foreign asset position) of the home country, $\bar{B}_t^F$, in percent of GDP in the entire economy, and is taken as given by the domestic household.[14]. $\Phi(.)$ introduces a convex cost that allows to obtain a well-defined steady state, and captures the costs of undertaking positions in the international asset market.[15]

A.2 Production Sector

The production of this economy is undertaken by three sectors. First, there is a final goods sector, that uses intermediate tradable inputs from both countries and operates under perfect competition, to produce the final tradable goods. This same sector also aggregates varieties of the nontradable goods to produce a final nontradable good that is sold to households. The second

[13] We follow Benigno (2001). Schmitt-Grohe and Uribe (2001) and Kollmann (2002) develop small open-economy models introducing the same cost to achieve stationarity. Heathcote and Perri (2001) also make a similar assumption in a two-country RBC model. Unlike, Benigno (2001) we consider real bonds rather than nominal.

[14] As Benigno, P.(2001) points it out, some restrictions on $\phi(.)$ are necessary: $\phi(0) = 1$; assumes the value 1 only if $B_{F,t} = 0$; differentiable; and decreasing in the neighborhood of zero.

[15] Another way to describe this cost is to assume the existence of intermediaries in the foreign asset market (which are owned by the foreign households) who can borrow and lend to households of country F at a rate $(1+r^*)$, but can borrow from and lend to households of country H at a rate $(1+r^*)\phi(.)$.

sector produces intermediate tradable goods, which are used as an input for the production of final goods both in the home and in the foreign country. The third sector produces nontradable goods, that are used as inputs in the production of the final nontradable good.

A.2.1 Final Goods Sector

The final tradable good is consumed by domestic households. This good is produced by a continuum of firms, each producing the same variety, labelled by Y_t^T, using intermediate home $\left(X_t^h\right)$ and foreign $\left(X_t^f\right)$ goods with the following technology:

$$Y_t^T = \left\{ \gamma_x^{1/\theta} \left(X_t^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta} \left(X_t^f\right)^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution between home-produced and foreign-produced imported intermediate goods, and γ_x is the share of home goods in the production function. We further assume symmetric home-bias in the composite of intermediate tradable goods. The corresponding composite of home and foreign intermediate tradable goods abroad is given by

$$Y_t^{T^*} = \left\{ (1-\gamma_x)^{1/\theta} \left(X_t^{h^*}\right)^{\frac{\theta-1}{\theta}} + \gamma_x^{1/\theta} \left(X_t^{f^*}\right)^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

X_t^h and X_t^f, that denote the amount of home and foreign intermediate tradable inputs to produce the final tradable good at home, are also Dixit-Stiglitz aggregates of all types of home and foreign final goods, with elasticity of substitution σ:

$$X_t^h \equiv \left[\int_0^1 X_t^h(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}$$

and

$$X_t^f \equiv \left[\int_0^1 X_t^f(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}$$

where $X_t^h(h)$ and $X_t^f(f)$ denote individual quantities from intermediate tradable goods producers at home and foreign. The equivalent quantities for foreign final tradable goods producers are $X_t^{h^*}(h)$ and $X_t^{f^*}(f)$. Optimizing conditions by final tradable goods producers deliver the following demand functions:

$$X_t^h(h) = \gamma_x \left(\frac{P_t^h(h)}{P_t^h}\right)^{-\sigma} \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T;\ X_t^{h^*}(h) = (1-\gamma_x) \left(\frac{P_t^{h^*}(h)}{P_t^{h^*}}\right)^{-\sigma} \left(\frac{P_t^{h^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

$$X_t^f(f) = (1-\gamma_x) \left(\frac{P_t^f(f)}{P_t^f}\right)^{-\sigma} \left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T;\ X_t^{f^*}(f) = \gamma_x \left(\frac{P_t^{f^*}(f)}{P_t^{f^*}}\right)^{-\sigma} \left(\frac{P_t^{f^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

where

$$P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}}, \; P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}.$$

and

$$P_t^T = \left[\gamma_x \left(P_t^h\right)^{1-\theta} + (1-\gamma_x)\left(P_t^f\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

We assume that the law of one price holds for intermediate inputs, such that $P_t^h(h) = P_t^{h^*}(h)S_t$, and $P_t^f(f) = P_t^{f^*}(f)S_t$, where S_t is the nominal exchange rate.

The production of the final nontradable good is given by:

$$Y_t^N \equiv \left[\int_0^1 X_t^N(n)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}}$$

where we assume the same elasticity $\sigma > 1$ than in the case of final tradable goods produced within country H. The price level for nontradables is

$$P_t^N \equiv \left[\int_0^1 p_t^N(n)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$$

A.2.2 Intermediate Non-Tradable Goods Sector

The intermediate non-tradable sector produces differentiated goods that are aggregated by final good producing firms, and ultimately used for final consumption by domestic households only. Each firm produces intermediate nontradable goods according to the following production function

$$Y_t^N(n) = A_t Z_t^N L_t^N(n) \tag{28}$$

where A_t is a labor augmenting aggregate world technology shock which has a unit root with drift, as in Galí and Rabanal (2005):

$$\log A_t = g + \log A_{t-1} + \varepsilon_t^a \tag{29}$$

This shock also affects the intermediate tradable sector production function. Hence, real variables in both countries grow at a rate g. Z_t^N is the country-specific productivity shock to the non-tradable sector at time t which evolves according to an AR(1) process in logs

$$\log Z_t^N = (1-\rho^N)\log(\bar{Z}^N) + \rho^{Z,N} \log Z_{t-1}^N + \varepsilon_t^{Z,N} \tag{30}$$

Firms in the non-tradable sector face a Calvo lottery when setting their prices. Each period, with probability $1-\alpha_N$, firms receive a stochastic signal that allows them to reset prices optimally.

We assume that there is partial indexation with a coefficient φ_N to last period's sectorial inflation rate for those firms that do not get to reset prices. As a result, firms maximize the following profits function:

$$Max_{P_t^N(n)} E_t \sum_{k=0}^{\infty} \alpha_N^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^N(n) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N}}{P_{t+k}} - MC_{t+k}^N \right] Y_{t+k}^{N,d}(n) \right\} \tag{31}$$

subject to

$$Y_{t+k}^{N,d}(n) = \left[\left(\frac{P_t^N(n)}{P_{t+k}^N} \right) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N} \right]^{-\sigma} Y_t^N \tag{32}$$

where $Y_t^{N,d}(n)$ is total individual demand for a given type of nontradable good n, and Y_t^N is aggregate demand for nontradable goods, as defined above. $\Lambda_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor, where $\lambda_t = \frac{\psi_t}{C_t - bC_{t-1}}$ is the marginal utility of consumption. MC_t^N corresponds to the real marginal cost in the non-tradable sector. From cost minimization:

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t}$$

A.2.3 Intermediate Tradable Goods Sector

The intermediate tradable sector produces differentiated goods that are sold to the final sector goods producers in the home and foreign countries. Most functional forms are similar to those presented for the nontradable sector.

Each firm produces tradable intermediate goods according to the following production function

$$Y_t^h(h) = A_t Z_t^h L_t^h(h) \tag{33}$$

where Z_t^h is the country-specific productivity shock to the intermediate goods tradable sector at time t which evolves according to an AR(1) process in logs

$$\log Z_t^h = (1-\rho^h)\log(\bar{Z}^h) + \rho^{Z,h} \log Z_{t-1}^h + \varepsilon_t^{Z,h} \tag{34}$$

Firms in the intermediate tradable sector face the same Calvo lottery as firms in the intermediate nontradable sector, with relevant parameters α_h and φ_h:

$$Max_{P_t^h(h)} E_t \sum_{k=0}^{\infty} \alpha_h^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^h(h) \left(\frac{P_{t+k-1}^h}{P_{t-1}^h} \right)^{\varphi_h}}{P_{t+k}} - MC_{t+k}^h \right] Y_{t+k}^{h,d}(h) \right\} \tag{35}$$

subject to

$$\begin{aligned} Y_{t+k}^{h,d}(h) &= X_{t+k}^{h}(h) + X_{t+k}^{h^*}(h) \\ &= \left[\left(\frac{P_t^h(h)}{P_{t+k}^h}\right)\left(\frac{P_{t+k-1}^h}{P_{t-1}^h}\right)^{\varphi_h}\right]^{-\sigma} X_t^h \end{aligned} \quad (36)$$

where $Y_t^{h,d}(h)$ is total individual demand for a given type of tradable intermediate good h, and X_t^h is aggregate demand for intermediate good h, consisting of home demand, and foreign demand:

$$X_t^h = \left[\gamma_x \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T + (1-\gamma_x)\left(\frac{P_t^{h^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}\right]$$

MC_t^h corresponds to the real marginal cost in the non-tradable sector. From cost minimization:

$$MC_t^h = \frac{W_t}{P_t Z_t^h A_t}$$

A.2.4 Market Clearing

We assume that government spending is allocated between tradable and nontradable goods in the same way that private consumption is. Hence the market clearing conditions for both types of final goods, consisting of private consumption and government spending, are:

$$Y_t^T = C_t^T + G_t^T$$

$$Y_t^N = C_t^N + G_t^N$$

where G_t^N, G_t^T follow AR(1) processes in logs.

For the nontradable intermediate goods, the market clearing condition is:

$$Y_t^N(n) = X_t^N, \text{ for all } n \in [0,1] \quad (37)$$

while for the intermediate tradable goods sector it is:

$$Y_t^h(h) = X_t^h(h) + X_t^{h^*}(h), \text{ for all } h \in [0,1] \quad (38)$$

For the labor market:

$$\begin{aligned} L_t &= L_t^h + L_t^N = \\ &= \int_0^1 L_t^h(h)dh + \int_0^1 L_t^N(n)dn \end{aligned} \quad (39)$$

A.3 Optimizing and Market Clearing Conditions, Fiscal and Monetary Policy

In this subsection we present the full set of equations characterizing the symmetric equilibrium. Since all agents in each economy are equal, then the per capita and aggregate consumption levels are equal ($C_t = \bar{C}_t$), as well as the net foreign assets levels ($B_t^F = \bar{B}_t^F$).

A.3.1 Households

The Euler equations for home and foreign households, and the optimal condition of holdings by home household of the foreign bond are:

$$
\begin{aligned}
\lambda_t &= \beta E_t \left\{ R_t \frac{P_t}{P_{t+1}} \lambda_{t+1} \right\} \\
\lambda_t^* &= \beta E_t \left\{ R_t^* \frac{P_t^*}{P_{t+1}^*} \lambda_{t+1}^* \right\} \\
\lambda_t &= \Phi \left(\frac{S_t B_t^F}{P_t Y_t} \right) \beta E_t \left\{ R_t^* \frac{Q_{t+1}}{Q_t} \lambda_{t+1} \right\}
\end{aligned}
$$

where λ_t is the marginal utility of consumption:

$$
\begin{aligned}
\lambda_t &= U_C(C_t) = \frac{\psi_t}{C_t - bC_{t-1}} \\
\lambda_t^* &= U_C(C_t^*) = \frac{\psi_t^*}{C_t^* - b^* C_{t-1}^*}
\end{aligned}
$$

The labor supply decisions in each country are:

$$\lambda_t \frac{W_t}{P_t} = L_t^{\varphi}$$

$$\lambda_t^* \frac{W_t^*}{P_t^*} = (L_t^*)^{\varphi^*}$$

where:

$$L_t = L_t^h + L_t^N$$

and

$$L_t^* = L_t^{h^*} + L_t^{N^*}$$

Household demand for final tradable and nontradable goods are given by:

$$
\begin{aligned}
C_t^T &= \gamma_c \left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t, \\
C_t^N &= (1-\gamma_c)\left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t. \\
C_t^{T*} &= \gamma_c^* \left(\frac{P_t^{T*}}{P_t^*}\right)^{-\varepsilon^*} C_t^*, \\
C_t^{N*} &= (1-\gamma_c^*)\left(\frac{P_t^{N*}}{P_t^*}\right)^{-\varepsilon} C_t^*
\end{aligned}
$$

and the CPI's in each country are given by:

$$
\begin{aligned}
P_t &\equiv \left[\gamma_c \left(P_t^T\right)^{1-\varepsilon} + (1-\gamma_c)\left(P_t^N\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \\
P_t^* &\equiv \left[\gamma_c^* \left(P_t^{T*}\right)^{1-\varepsilon} + (1-\gamma_c^*)\left(P_t^{N*}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

The real exchange rate is

$$
Q_t = \frac{S_t P_t^*}{P_t}
$$

A.3.2 Final Goods Producers

The production of final tradable goods in both countries is given by:

$$
Y_t^T = \left\{\gamma_x^{1/\theta}\left(X_t^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta}\left(X_t^f\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}
$$

and

$$
Y_t^{T*} = \left\{(1-\gamma_x)^{1/\theta}\left(X_t^{h*}\right)^{\frac{\theta-1}{\theta}} + \gamma_x^{1/\theta}\left(X_t^{f*}\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}
$$

Demand for intermediate tradable goods is:

$$
X_t^h = \gamma_x \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T;\; X_t^{h*} = (1-\gamma_x)\left(\frac{P_t^{h*}}{P_t^{T*}}\right)^{-\theta} Y_t^{T*}
$$

$$
X_t^f = (1-\gamma_x)\left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T;\; X_t^{f*}(f) = \gamma_x \left(\frac{P_t^{f*}}{P_t^{T*}}\right)^{-\theta} Y_t^{T*}
$$

where

$$
P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}},\; P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}.
$$

The price of final tradable goods is:

$$P_t^T = \left[\gamma_x \left(P_t^h \right)^{1-\theta} + (1-\gamma_x) \left(P_t^f \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

and

$$P_t^{T^*} = \left[\gamma_x^* \left(P_t^{h^*} \right)^{1-\theta} + (1-\gamma_x^*) \left(P_t^{f^*} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Since we assumed that the law of one price holds for intermediate goods, it also holds in the aggregate, such that $P_t^h = P_t^{h^*} S_t$, and $P_t^f = P_t^{f^*} S_t$, where S_t is the nominal exchange rate.

A.3.3 Nontradable Goods Producers

The price setting equations are given by the following optimal expressions:

$$\frac{\hat{p}_t^N}{P_t^N} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N} \right)^{-\sigma} MC_{t+k}^N Y_{t+k}^N}{\sum_{k=0}^{\infty} \beta^k \alpha_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N} \right)^{1-\sigma} \frac{P_{t+k}^N}{P_{t+k}} Y_{t+k}^N} \right\}$$

where

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t},$$

$$Y_t^N = C_t^N + G_t^N$$

The evolution of the price level of nontradables is

$$P_t^N \equiv \left[\alpha_N \left(P_{t-1}^N \left(\Pi_{t-1}^N \right)^{\varphi_N} \right)^{1-\sigma} + (1-\alpha_N) \left(\hat{p}_t^N \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$.

The production function is:

$$Y_t^N = A_t Z_t^N L_t^N.$$

In the foreign country these expressions are:

$$\frac{\hat{p}_t^{N^*}}{P_t^{N^*}} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_{N^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}}{\Pi_{t+s}^{N^*}} \right)^{-\sigma} MC_{t+k}^{N^*} Y_{t+k}^{N^*}}{\sum_{k=0}^{\infty} \beta^k \alpha_{N^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}}{\Pi_{t+s}^{N^*}} \right)^{1-\sigma} \frac{P_{t+k}^{N^*}}{P_{t+k}} Y_{t+k}^{N^*}} \right\}$$

where

$$MC_t^{N^*} = \frac{W_t}{P_t Z_t^{N^*} A_t},$$
$$Y_t^{N^*} = C_t^{N^*} + G_t^{N^*}$$

The evolution of the price level of nontradables is

$$P_t^{N^*} \equiv \left[\alpha_{N^*} \left(P_{t-1}^{N^*} \left(\Pi_{t-1}^{N^*} \right)^{\varphi_{N^*}} \right)^{1-\sigma} + (1-\alpha_{N^*}) \left(\hat{p}_t^{N^*} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^{N^*} = \frac{P_{t-1}^{N^*}}{P_{t-2}^{N^*}}$.

The production function is:

$$Y_t^{N^*} = A_t Z_t^{N^*} L_t^{N^*}.$$

A.3.4 Intermediate Traded Goods Producers

The price setting equations are given by the following optimal expressions:

$$\frac{\hat{p}_t^h}{P_t^h} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_h^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^h\right)^{\varphi_h}}{\Pi_{t+s}^h} \right)^{-\sigma} MC_{t+k}^h Y_{t+k}^h}{\sum_{k=0}^{\infty} \beta^k \alpha_h^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^h\right)^{\varphi_h}}{\Pi_{t+s}^h} \right)^{1-\sigma} \frac{P_{t+k}^h}{P_{t+k}} Y_{t+k}^h} \right\}$$

where

$$MC_t^h = \frac{W_t}{P_t Z_t^h A_t},$$
$$Y_t^h = X_t^h + X_t^{h^*}.$$

The evolution of the price level of final tradables is

$$P_t^h \equiv \left[\alpha_h \left(P_{t-1}^h \left(\Pi_{t-1}^h \right)^{\varphi_h} \right)^{1-\sigma} + (1-\alpha_h) \left(\hat{p}_t^h \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^h = \frac{P_{t-1}^h}{P_{t-2}^h}$.

The production function is:

$$Y_t^h = A_t Z_t^h L_t^h$$

In the foreign country, this expressions are:

$$\frac{\hat{p}_t^{f^*}}{P_t^{f^*}} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_{f^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{f^*}\right)^{\varphi_{f^*}}}{\Pi_{t+s}^{f^*}} \right)^{-\sigma} MC_{t+k}^{f^*} Y_{t+k}^{f^*}}{\sum_{k=0}^{\infty} \beta^k \alpha_{f^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{f^*}\right)^{\varphi_{f^*}}}{\Pi_{t+s}^{f^*}} \right)^{1-\sigma} \frac{P_{t+k}^{f^*}}{P_{t+k}} Y_{t+k}^{f^*}} \right\}$$

where

$$MC_t^{f^*} = \frac{W_t}{P_t Z_t^{f^*} A_t},$$

$$Y_t^{f^*} = X_t^f + X_t^{f^*}.$$

The evolution of the price level of final tradables is

$$P_t^{f^*} \equiv \left[\alpha_{f^*}\left(P_{t-1}^{f^*}\left(\Pi_{t-1}^{f^*}\right)^{\varphi_{f^*}}\right)^{1-\sigma} + (1-\alpha_{f^*})\left(\hat{p}_t^{f^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^{f^*} = \frac{P_{t-1}^{f^*}}{P_{t-2}^{f^*}}$.

The production function is:

$$Y_t^{f^*} = A_t Z_t^{f^*} L_t^{f^*}$$

A.3.5 Monetary Policy

Monetary policy in both countries is conducted with a Taylor rule that targets CPI inflation and output growth deviation from steady-state values:

$$R_t = \bar{R}^{(1-\rho_r)} R_{t-1}^{\rho_r} \left(\frac{P_t/P_{t-1}}{\Pi}\right)^{(1-\rho_r)\gamma_\pi} \left(\frac{Y_t/Y_{t-1}}{1+g}\right)^{(1-\rho_r)\gamma_y} \exp(\varepsilon_t^r)$$

$$R_t^* = \bar{R}^{*(1-\rho_r^*)} \left(R_{t-1}^*\right)^{\rho_r^*} \left(\frac{P_t^*/P_{t-1}^*}{\Pi^*}\right)^{(1-\rho_r^*)\gamma_\pi^*} \left(\frac{Y_t^*/Y_{t-1}^*}{1+g}\right)^{(1-\rho_r^*)\gamma_y^*} \exp(\varepsilon_t^{r^*})$$

A.3.6 Fiscal Policy

Fiscal policy is conducted with lump-sum taxation and exogenous government spending:

$$G_t^T = (\bar{G}^T)^{(1-\rho_{G^T})}(G_{t-1}^T)^{\rho_{G^T}} \exp(\varepsilon_t^{G^T})$$

$$G_t^N = (\bar{G}^N)^{(1-\rho_{G^N})}(G_{t-1}^N)^{\rho_{G^N}} \exp(\varepsilon_t^{G^N})$$

$$G_t^{T^*} = (\bar{G}^{T^*})^{(1-\rho_{G^{T^*}})}(G_{t-1}^{T^*})^{\rho_{G^{T^*}}} \exp(\varepsilon_t^{G^{T^*}})$$

$$G_t^{N^*} = (\bar{G}^{N^*})^{(1-\rho_{G^{N^*}})}(G_{t-1}^{N^*})^{\rho_{G^{N^*}}} \exp(\varepsilon_t^{G^{N^*}})$$

A.3.7 Trade Balance and Net Foreign Asset Dynamics

We present the evolution of the trade balance and net foreign assets of the home country, since the definition those in the foreign country will mirror those in the home country. Holdings of foreign bonds depend on the trade balance (NX_t) as follows

$$\frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t B_t^F}{P_t Y_t}\right)} = \frac{S_t B_{t-1}^F}{P_t} + NX_t$$

Since international trade only occurs at the intermediate goods level, net exports equal exports minus imports of intermediate goods:

$$NX_t = \frac{P_t^h X_t^{h^*} - P_t^f X_t^f}{P_t}$$

Finally, we define nominal GDP to be equal to aggregate nominal private and public consumption, hence $P_t Y_t = P_t^T (C_t^T + G_t^T) + P_t^N (C_t^N + G_t^N)$.

B Appendix: Log-Linear version of the Model

Euler equations

$$b\Delta c_t = -(1+g-b)(r_t - E_t\triangle p_{t+1}) + (1+g)E_t\Delta c_{t+1} + (1+g-b)\left(1-\rho_\psi\right)\widehat{\psi}_t \tag{40}$$

$$b^*\Delta c_t^* = -(1+g-b^*)\left(r_t^* - E_t\triangle p_{t+1}^*\right) + (1+g)E_t\Delta c_{t+1}^* + (1+g-b^*)\left(1-\rho_\psi^*\right)\widehat{\psi}_t^* \tag{41}$$

Risk sharing

$$\begin{aligned} E_t(q_{t+1}-q_t) &= \left[\frac{(1+g)E_t\Delta c_{t+1} - b\Delta c_t}{(1+g-b)}\right] - \left[\frac{(1+g)E_t\Delta c_{t+1}^* - b^*\Delta c_t^*}{(1+g-b^*)}\right] \\ &\quad + \left(1-\rho_\psi\right)\widehat{\psi}_t - \left(1-\rho_\psi^*\right)\widehat{\psi}_t^* + \chi b_t \end{aligned} \tag{42}$$

where $\chi \equiv -\Phi'(0)\,Y$, $b_t = \left(\frac{S_t B_t^F}{P_t}\right)Y^{-1}$.

The labor supply schedules are given by:

$$\tilde{\omega}_t = \varphi l_t + \left[\frac{1+g}{1+g-b}\right]\widetilde{c}_t - \frac{b}{(1+g-b)}\widetilde{c}_{t-1} + \frac{b}{(1+g-b)}\varepsilon_t^a \tag{43}$$

$$\tilde{\omega}_t^* = \varphi l_t^* + \left[\frac{1+g}{1+g-b^*}\right]\widetilde{c}_t^* - \frac{b^*}{(1+g-b^*)}\widetilde{c}_{t-1}^* + \frac{b^*}{(1+g-b^*)}\varepsilon_t^a \tag{44}$$

Technology

$$\widetilde{y}_t^h = l_t^h + z_t^h - \varepsilon_t^a \tag{45}$$

$$\widetilde{y}_t^{h^*} = l_t^{h^*} + z_t^{h^*} - \varepsilon_t^a \tag{46}$$

$$\widetilde{y}_t^N = l_t^N + z_t^N - \varepsilon_t^a \tag{47}$$

$$\widetilde{y}_t^{N^*} = l_t^{N^*} + z_t^{N^*} - \varepsilon_t^a \tag{48}$$

Consumer price inflation

$$\triangle p_t = \gamma_c\triangle p_t^T + (1-\gamma_c)\triangle p_t^N \tag{49}$$

$$\triangle p_t = \gamma_{c^*}\triangle p_t^{T^*} + (1-\gamma_{c^*})\triangle p_t^{N^*} \tag{50}$$

Tradable inflation

$$\triangle p_t^T = \gamma_x\triangle p_t^h + (1-\gamma_x)(\triangle p_t^{f^*} + \Delta s_t) \tag{51}$$

$$\triangle p_t^{T^*} = \gamma_{x^*}(\triangle p_t^h - \Delta s_t) + (1-\gamma_{x^*})\triangle p_t^{f^*} \tag{52}$$

Price setting in the non-tradable sector

$$\triangle p_t^N - \varphi_N\triangle p_{t-1}^N = \beta E_t\left(\triangle p_{t+1}^N - \varphi_N\triangle p_t^N\right) + \kappa_N\left(\widetilde{w}_t - z_t^N - t_t^N\right) \tag{53}$$

$$\triangle p_t^{N^*} - \varphi_{N^*} \triangle p_{t-1}^{N^*} = \beta E_t \left(\triangle p_{t+1}^{N^*} - \varphi_N^* \triangle p_t^{N^*} \right) + \kappa_{N^*} \left(\widetilde{w}_t^* - z_t^{N^*} - t_t^{N^*} \right) \tag{54}$$

where $\kappa_N = (1-\alpha_N)(1-\beta\alpha_N)/\alpha_N$, $\kappa_{N^*} = (1-\alpha_{N^*})(1-\beta\alpha_{N^*})/\alpha_{N^*}$, $t^N = p_t^N - p_t$, and $t^{N^*} = p_t^{N^*} - p_t$.

Price Setting in the Intermediate tradable good sector

$$\triangle p_t^h - \varphi_h \triangle p_{t-1}^h = \beta E_t \left(\triangle p_{t+1}^h - \varphi_h \triangle p_t^h \right) + \kappa_h \left(\widetilde{w}_t - z_t^h - t_t^h - t_t^T \right) \tag{55}$$

$$\triangle p_t^{f^*} - \varphi_{f^*} \triangle p_{t-1}^{f^*} = \beta E_t \left(\triangle p_{t+1}^{f^*} - \varphi_f^* \triangle p_t^{f^*} \right) + \kappa_{f^*} \left(\widetilde{w}_t^* - z_t^{f^*} - t_t^{f^*} - t_t^{T^*} \right) \tag{56}$$

where $\kappa_h = (1-\alpha_h)(1-\beta\alpha_h)/\alpha_h$, $\kappa_{f^*} = (1-\alpha_{f^*})(1-\beta\alpha_{f^*})/\alpha_{f^*}$, $t_t^h = p_t^h - p_t^T$, $t_t^{f^*} = p_t^{f^*} - p_t^{T^*}$, $t^T = p_t^T - p_t$, and $t^{T^*} = p_t^{T^*} - p_t$.

Final consumption demand

$$\widetilde{c}_t^T = -\varepsilon t_t^T + \widetilde{c}_t \tag{57}$$

$$\widetilde{c}_t^{T^*} = -\varepsilon^* t_t^{T^*} + \widetilde{c}_t^* \tag{58}$$

$$\widetilde{c}_t^N = -\varepsilon t_t^N + \widetilde{c}_t \tag{59}$$

$$\widetilde{c}_t^{N^*} = -\varepsilon^* t_t^{N^*} + \widetilde{c}_t^* \tag{60}$$

Intermediate tradable and nontradable demand

$$\widetilde{x}_t^h = -\theta t_t^h + \widetilde{y}_t^T \tag{61}$$

$$\widetilde{x}_t^{h^*} = -\theta t_t^{h^*} + \widetilde{y}_t^{T^*} \tag{62}$$

$$\widetilde{x}_t^f = -\theta t_t^f + \widetilde{y}_t^T \tag{63}$$

$$\widetilde{x}_t^{f^*} = -\theta t_t^{f^*} + \widetilde{y}_t^{T^*} \tag{64}$$

Relative Price Index

Let´s define $t_t = \frac{p_t^f}{p_t^h}$, since the law of one price holds $t_t = -t_t^* = \frac{p_t^{h^*}}{p_t^{f^*}}$, then we can write the following relative prices as a function of t_t (as in Rabanal and Tuesta 2006, Benigno and Benigno and others).

$$t_t^h = -(1-\gamma_x)\, t_t \tag{65}$$

$$t_t^{h^*} = -\gamma_x t_t \tag{66}$$

$$t_t^f = \gamma_x t_t \tag{67}$$

$$t_t^{f^*} = (1-\gamma_x)\, t_t \tag{68}$$

Relative prices

$$t_t = t_{t-1} + \triangle s_t + \triangle p_t^{f^*} - \triangle p_t^h \tag{69}$$

$$t_t^T = t_{t-1}^T + \triangle p_t^T - \triangle p_t \tag{70}$$

$$t_t^N = -\frac{\gamma_c}{1-\gamma_c} t_t^T \tag{71}$$

$$t_t^{T^*} = t_{t-1}^{T^*} + \triangle p_t^{T^*} - \triangle p_t^* \tag{72}$$

$$t_t^{N^*} = -\frac{\gamma_c^*}{1-\gamma_c^*} t_t^{T^*} \tag{73}$$

$$q_t = q_{t-1} + \triangle s_t + \triangle p_t^* - \triangle p_t \tag{74}$$

Taylor rules

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\, \gamma_\pi \triangle p_t + \gamma_y \triangle y_t + \varepsilon_t^r \tag{75}$$

$$r_t^* = \rho_r^* r_{t-1}^* + (1-\rho_{r^*})\, \gamma_\pi^* \triangle p_t^* + \gamma_y^* \triangle y_t + \varepsilon_t^{r^*} \tag{76}$$

Net Foreign Assets and Net exports

$$\beta \widetilde{b}_t - \frac{1}{1+g} \widetilde{b}_{t-1} = \widetilde{nx}_t \tag{77}$$

$$\widetilde{nx}_t = \frac{X^f}{Y}\left(\widetilde{x}_t^{h^*} - \widetilde{x}_t^f - t_t\right) \tag{78}$$

where $\widetilde{b}_t = \frac{B_t^F S_t}{Y P_t}$ is the debt to GDP ratio, and $\widetilde{nx}_t = \frac{NX_t}{Y}$, and where we have assumed balanced trade in the steady state. To solve for the steady-state ratios, we have that:

$$\frac{Y^T}{Y} = \frac{G^T}{G} = \frac{C^T}{C} = \gamma_c$$

$$\frac{Y^N}{Y} = \frac{G^N}{G} = \frac{C^N}{C} = 1-\gamma_c$$

$$\frac{X^h}{Y^T} = \gamma_x, \; \frac{X^f}{Y^T} = 1-\gamma_x$$

Therefore tradable GDP over total GDP is

$$\frac{X^h}{Y} = \frac{X^h}{Y^T}\frac{Y^T}{Y} = \gamma_x \gamma_c.$$

Hence

$$\frac{X^f}{Y} = \frac{X^f}{X^T}\frac{X^T}{Y^T}\frac{Y^T}{Y} = (1-\gamma_x)\gamma_c.$$

Market clearing:

$$\tilde{y}_t^T = (1-\gamma)\tilde{c}_t^T + \gamma \tilde{g}_t^T \tag{79}$$

where γ equals the fraction of government spending over total output. For the nontradable good, the market clearing condition is:

$$\tilde{y}_t^N = (1-\gamma)\tilde{c}_t^N + \gamma\tilde{g}_t^N \tag{80}$$

Finally, for the intermediate tradable goods sector:

$$\widetilde{y}_t^h = \gamma_x \widetilde{x}_t^h + (1-\gamma_x)\,\widetilde{x}_t^{h^*} \tag{81}$$

Total real GDP

$$\widetilde{y}_t = \gamma_c(t_t^T + \widetilde{y}_t^T) + (1-\gamma_c)\,(t_t^N + \widetilde{y}_t^N) \tag{82}$$

total labor

$$l_t = \frac{X^h}{X^h + Y^N} l_t^h + \frac{Y^N}{X^h + Y^N} l_t^N \tag{83}$$

For the foreign country:

$$\tilde{y}_t^{T^*} = (1-\gamma^*)\tilde{c}_t^{T^*} + \gamma^*\tilde{g}_t^{T^*} \tag{84}$$

$$\tilde{y}_t^{N^*} = (1-\gamma^*)\tilde{c}_t^{N^*} + \gamma^*\tilde{g}_t^{N^*} \tag{85}$$

$$\widetilde{y}_t^{f^*} = \gamma_x^* \widetilde{x}_t^{f^*} + (1-\gamma_x^*)\,\widetilde{x}_t^{f} \tag{86}$$

$$\widetilde{y}_t^* = \gamma_c^*(t_t^{T^*} + \widetilde{y}_t^{T^*}) + (1-\gamma_c^*)\,(t_t^{N^*} + \widetilde{y}_t^{N^*}) \tag{87}$$

$$l_t = \frac{Y^{f^*}}{Y^h + Y^N} l_t^{f^*} + \frac{Y^{N^*}}{Y^{f^*} + Y^{N^*}} l_t^{N*} \tag{88}$$

Mapping variables in the model with observable variables.

$$\widetilde{c}_t - \widetilde{c}_{t-1} = \triangle c_t - \varepsilon_t^a \tag{89}$$

$$\widetilde{c}_t^* - \widetilde{c}_{t-1}^* = \triangle c_t^* - \varepsilon_t^a \tag{90}$$

$$\widetilde{y}_t - \widetilde{y}_{t-1} = \triangle y_t - \varepsilon_t^a \tag{91}$$

$$\widetilde{y}_t^* - \widetilde{y}_{t-1}^* = \triangle y_t^* - \varepsilon_t^a \tag{92}$$

Shocks

$$\psi_t = \rho_\psi \psi_{t-1} + \varepsilon_t^\psi$$

$$\psi_t^* = \rho_{\psi^*} \psi_{t-1}^* + \varepsilon_t^{\psi^*}$$

$$z_t^h = \rho^{Z,h} z_{t-1}^h + \varepsilon_t^{Z,h}$$

$$z_t^{f^*} = \rho^{Z,f^*} z_{t-1}^{f^*} + \varepsilon_t^{Z,f^*}$$

$$z_t^N = \rho^{Z,N} z_{t-1}^N + \varepsilon_t^{Z,N}$$
$$z_t^{N^*} = \rho^{Z,N^*} z_{t-1}^{N^*} + \varepsilon_t^{Z,N^*}$$
$$g_t^T = \rho^{G,T} g_{t-1}^T + \varepsilon_t^{G,T}$$
$$g_t^N = \rho^{G,N} g_{t-1}^N + \varepsilon_t^{G,N}$$
$$g_t^{T^*} = \rho^{G,T^*} g_{t-1}^{T^*} + \varepsilon_t^{G,T^*}$$
$$g_t^{N^*} = \rho^{G,N^*} g_{t-1}^{N^*} + \varepsilon_t^{G,N^*}$$

and $\varepsilon_t^a, \varepsilon_t^r, \varepsilon_t^{r^*}$ are iid shocks.