Nontradable Goods and the Real Exchange Rate

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Abstract

This online appendix provides the equilibrium conditions of the model and the loglinear approximation.

A Appendix: The Baseline Model

In this appendix, we present the full version of a model with tradable and nontradable final consumption goods, in the spirit of Stockman and Tesar (1995) and Dotsey and Duarte (2008). We introduce sticky prices in both sectors to be able to study inflation dynamics and their role in affecting the real exchange rate.

A.1 Households

A.1.1 Preferences

Representative households in the home country are assumed to maximize the following utility function:

$$U_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \psi_t \left[\log \left(C_t - b\bar{C}_{t-1} \right) - \frac{L_t^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{1}$$

subject to the following budget constraint:

$$\frac{B_t^H}{P_t R_t} + \frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t \bar{B}_t^F}{P_t Y_t}\right)} \leq \frac{B_{t-1}^H}{P_t} + \frac{S_t B_{t-1}^F}{P_t} + \frac{W_t}{P_t} L_t - C_t + \Pi_t \tag{2}$$

E_0 denotes the conditional expectation on information available at date $t = 0$, β is the intertemporal discount factor, with $0 < \beta < 1$. C_t denotes the level of consumption in period t, L_t denotes labor supply. The utility function displays external habit formation with respect to the habit stock, which is last period's aggregate consumption of the economy $\bar{C}_{t-1}$. $b \in [0, 1]$ denotes the importance of the habit stock. $\varphi > 0$ is inverse elasticity of labor supply with respect to the real wage. ψ_t is a preference shock that follows an $AR(1)$ process in logs

$$\log \psi_t = \rho_\psi \log \psi_{t-1} + \varepsilon_t^\psi \tag{3}$$

We define the consumption index as

$$C_t \equiv \left[\gamma_c^{1/\varepsilon} \left(C_t^T \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma_c)^{1/\varepsilon} \left(C_t^N \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is elasticity of substitution between the final tradable (C_t^T) and final nontradable (C_t^N) goods, and γ_c is the share of final tradable goods in the consumption basket at home.

In this context, the consumer price index that corresponds to the previous specification is given by

$$P_t \equiv \left[\gamma_c \left(P_t^T \right)^{1-\varepsilon} + (1-\gamma_c) \left(P_t^N \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}},$$

where all prices are for goods sold in the home country, in home currency and at consumer level, for both tradable and nontradable goods.

Demands for the final tradable and nontradable goods are given by:

$$\begin{aligned} C_t^T &= \gamma_c \left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t, \\ C_t^N &= (1-\gamma_c)\left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t. \end{aligned}$$

A.1.2 Incomplete Asset Markets

For modelling simplicity, we choose to model incomplete markets with two risk-free one-period nominal bonds denominated in domestic and foreign currency, and a cost of bond holdings is introduced to achieve stationarity. Then, the budget constraint of the domestic households in real units of home currency is given by:

$$\frac{B_t^H}{P_t R_t} + \frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t B_t^F}{P_t Y_t}\right)} \leq \frac{B_{t-1}^H}{P_t} + \frac{S_t B_{t-1}^F}{P_t} + \frac{W_t}{P_t} L_t - C_t + \Pi_t \tag{4}$$

where W_t is the nominal wage, and Π_t are real profits for the home consumer. B_t^H is the holding of the risk free domestic nominal bond and B_t^F is the holding of the foreign risk-free nominal bond expressed in foreign country currency. S_t is the nominal exchange rate, expressed in units of home country currency per unit of foreign country. The function $\Phi(.)$ depends on the net liability position (i.e. the negative net foreign asset position) of the home country, $\bar{B}_t^F$, in percent of GDP in the entire economy, and is taken as given by the domestic household.[1] $\Phi(.)$ introduces a convex cost that allows to obtain a well-defined steady state, and captures the costs of undertaking positions in the international asset market.[2]

A.2 Production Sector

The production of this economy is undertaken by three sectors. First, there is a final goods sector, that uses intermediate tradable inputs from both countries and operates under perfect competition, to produce the final tradable goods. This same sector also aggregates varieties of the nontradable goods to produce a final nontradable good that is sold to households. The second

[1] As Benigno, P.(2001) points it out, some restrictions on $\phi(.)$ are necessary: $\phi(0) = 1$; assumes the value 1 only if $B_{F,t} = 0$; differentiable; and decreasing in the neighborhood of zero.

[2] Another way to describe this cost is to assume the existence of intermediaries in the foreign asset market (which are owned by the foreign households) who can borrow and lend to households of country F at a rate $(1+r^*)$, but can borrow from and lend to households of country H at a rate $(1+r^*)\phi(.)$.

sector produces intermediate tradable goods, which are used as an input for the production of final goods both in the home and in the foreign country. The third sector produces nontradable goods, that are used as inputs in the production of the final nontradable good.

A.2.1 Final Goods Sector

The final tradable good is consumed by domestic households. This good is produced by a continuum of firms, each producing the same variety, labelled by Y_t^T, using intermediate home $\left(X_t^h\right)$ and foreign $\left(X_t^f\right)$ goods with the following technology:

$$Y_t^T = \left\{ \gamma_x^{1/\theta} \left(X_t^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta} \left(X_t^f\right)^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

where θ is the elasticity of substitution between home-produced and foreign-produced imported intermediate goods, and γ_x is the share of home goods in the production function. We further assume symmetric home-bias in the composite of intermediate tradable goods. The corresponding composite of home and foreign intermediate tradable goods abroad is given by

$$Y_t^{T^*} = \left\{ (1-\gamma_x)^{1/\theta} \left(X_t^{h^*}\right)^{\frac{\theta-1}{\theta}} + \gamma_x^{1/\theta} \left(X_t^{f^*}\right)^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

X_t^h and X_t^f, that denote the amount of home and foreign intermediate tradable inputs to produce the final tradable good at home, are also Dixit-Stiglitz aggregates of all types of home and foreign final goods, with elasticity of substitution σ:

$$X_t^h \equiv \left[\int_0^1 X_t^h(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}$$

and

$$X_t^f \equiv \left[\int_0^1 X_t^f(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}$$

where $X_t^h(h)$ and $X_t^f(f)$ denote individual quantities from intermediate tradable goods producers at home and foreign. The equivalent quantities for foreign final tradable goods producers are $X_t^{h^*}(h)$ and $X_t^{f^*}(f)$. Optimizing conditions by final tradable goods producers deliver the following demand functions:

$$X_t^h(h) = \gamma_x \left(\frac{P_t^h(h)}{P_t^h}\right)^{-\sigma} \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T; \; X_t^{h^*}(h) = (1-\gamma_x) \left(\frac{P_t^{h^*}(h)}{P_t^{h^*}}\right)^{-\sigma} \left(\frac{P_t^{h^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

$$X_t^f(f) = (1-\gamma_x) \left(\frac{P_t^f(f)}{P_t^f}\right)^{-\sigma} \left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T; \; X_t^{f^*}(f) = \gamma_x \left(\frac{P_t^{f^*}(f)}{P_t^{f^*}}\right)^{-\sigma} \left(\frac{P_t^{f^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}$$

where

$$P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}}, \; P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}.$$

and

$$P_t^T = \left[\gamma_x \left(P_t^h\right)^{1-\theta} + (1-\gamma_x)\left(P_t^f\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

We assume that the law of one price holds for intermediate inputs, such that $P_t^h(h) = P_t^{h^*}(h)S_t$, and $P_t^f(f) = P_t^{f^*}(f)S_t$, where S_t is the nominal exchange rate.

The production of the final nontradable good is given by:

$$Y_t^N \equiv \left[\int_0^1 X_t^N(n)^{\frac{\sigma-1}{\sigma}} dn\right]^{\frac{\sigma}{\sigma-1}}$$

where we assume the same elasticity $\sigma > 1$ than in the case of final tradable goods produced within country H. The price level for nontradables is

$$P_t^N \equiv \left[\int_0^1 p_t^N(n)^{1-\sigma} dn\right]^{\frac{1}{1-\sigma}}$$

A.2.2 Intermediate Non-Tradable Goods Sector

The intermediate nontradable sector produces differentiated goods that are aggregated by final good producing firms, and ultimately used for final consumption by domestic households only. Each firm produces intermediate nontradable goods according to the following production function

$$Y_t^N(n) = A_t Z_t^N L_t^N(n) \tag{5}$$

where A_t is a labor augmenting aggregate world technology shock which has a unit root with drift, as in Galí and Rabanal (2005):

$$\log A_t = g + \log A_{t-1} + \varepsilon_t^a \tag{6}$$

This shock also affects the intermediate tradable sector production function. Hence, real variables in both countries grow at a rate g. Z_t^N is the country-specific productivity shock to the nontradable sector at time t which evolves according to an AR(1) process in logs

$$\log Z_t^N = (1-\rho^N)\log(\bar{Z}^N) + \rho^{Z,N}\log Z_{t-1}^N + \varepsilon_t^{Z,N} \tag{7}$$

Firms in the nontradable sector face a Calvo lottery when setting their prices. Each period, with probability $1-\alpha_N$, firms receive a stochastic signal that allows them to reset prices optimally.

We assume that there is partial indexation with a coefficient φ_N to last period's sectorial inflation rate for those firms that do not get to reset prices. As a result, firms maximize the following profits function:

$$Max_{P_t^N(n)} E_t \sum_{k=0}^{\infty} \alpha_N^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^N(n) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N}}{P_{t+k}} - MC_{t+k}^N \right] Y_{t+k}^{N,d}(n) \right\} \tag{8}$$

subject to

$$Y_{t+k}^{N,d}(n) = \left[\left(\frac{P_t^N(n)}{P_{t+k}^N} \right) \left(\frac{P_{t+k-1}^N}{P_{t-1}^N} \right)^{\varphi_N} \right]^{-\sigma} Y_t^N \tag{9}$$

where $Y_t^{N,d}(n)$ is total individual demand for a given type of nontradable good n, and Y_t^N is aggregate demand for nontradable goods, as defined above. $\Lambda_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor, where $\lambda_t = \frac{\psi_t}{C_t - bC_{t-1}}$ is the marginal utility of consumption. MC_t^N corresponds to the real marginal cost in the nontradable sector. From cost minimization:

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t}$$

A.2.3 Intermediate Tradable Goods Sector

The intermediate tradable sector produces differentiated goods that are sold to the final sector goods producers in the home and foreign countries. Most functional forms are similar to those presented for the nontradable sector.

Each firm produces tradable intermediate goods according to the following production function

$$Y_t^h(h) = A_t Z_t^h L_t^h(h) \tag{10}$$

where Z_t^h is the country-specific productivity shock to the intermediate goods tradable sector at time t which evolves according to an AR(1) process in logs

$$\log Z_t^h = (1-\rho^h)\log(\bar{Z}^h) + \rho^{Z,h} \log Z_{t-1}^h + \varepsilon_t^{Z,h} \tag{11}$$

Firms in the intermediate tradable sector face the same Calvo lottery as firms in the intermediate nontradable sector, with relevant parameters α_h and φ_h:

$$Max_{P_t^h(h)} E_t \sum_{k=0}^{\infty} \alpha_h^k \Lambda_{t,t+k} \left\{ \left[\frac{P_t^h(h) \left(\frac{P_{t+k-1}^h}{P_{t-1}^h} \right)^{\varphi_h}}{P_{t+k}} - MC_{t+k}^h \right] Y_{t+k}^{h,d}(h) \right\} \tag{12}$$

subject to

$$
\begin{aligned}
Y_{t+k}^{h,d}(h) &= X_{t+k}^{h}(h) + X_{t+k}^{h^*}(h) \\
&= \left[\left(\frac{P_t^h(h)}{P_{t+k}^h} \right) \left(\frac{P_{t+k-1}^h}{P_{t-1}^h} \right)^{\varphi_h} \right]^{-\sigma} X_t^h
\end{aligned}
\tag{13}
$$

where $Y_t^{h,d}(h)$ is total individual demand for a given type of tradable intermediate good h, and X_t^h is aggregate demand for intermediate good h, consisting of home demand, and foreign demand:

$$
X_t^h = \left[\gamma_x \left(\frac{P_t^h}{P_t^T} \right)^{-\theta} Y_t^T + (1-\gamma_x) \left(\frac{P_t^{h^*}}{P_t^{T^*}} \right)^{-\theta} Y_t^{T^*} \right]
$$

MC_t^h corresponds to the real marginal cost in the nontradable sector. From cost minimization:

$$
MC_t^h = \frac{W_t}{P_t Z_t^h A_t}
$$

A.2.4 Market Clearing

We assume that the demand shock is allocated between tradable and nontradable goods in the same way that private consumption is. Hence the market clearing conditions for both types of final goods, consisting of private consumption and the demand shock in the tradadable sector, are:

$$
Y_t^T = C_t^T + G_t^T
$$

$$
Y_t^N = C_t^N + G_t^N
$$

where G_t^N, G_t^T follow AR(1) processes in logs. The bond market clearing conditions are

$$
B_t^H + B_t^{H^*} = 0 \tag{14}
$$

$$
B_t^F + B_t^{F^*} = 0 \tag{15}
$$

For the nontradable intermediate goods, the market clearing condition is:

$$
Y_t^N(n) = X_t^N, \text{ for all } n \in [0,1] \tag{16}
$$

while for the intermediate tradable goods sector it is:

$$
Y_t^h(h) = X_t^h(h) + X_t^{h^*}(h), \text{ for all } h \in [0,1] \tag{17}
$$

For the labor market:

$$
\begin{aligned}
L_t &= L_t^h + L_t^N = \\
&= \int_0^1 L_t^h(h)dh + \int_0^1 L_t^N(n)dn
\end{aligned}
\tag{18}
$$

A.3 Optimizing, Market Clearing Conditions, and Monetary Policy

In this subsection we present the full set of equations characterizing the symmetric equilibrium. Since all agents in each economy are equal, then the per capita and aggregate consumption levels are equal ($C_t = \bar{C}_t$), as well as the net foreign assets levels ($B_t^F = \bar{B}_t^F$).

A.3.1 Households

The Euler equations for home and foreign households, and the optimal condition of holdings by home household of the foreign bond are:

$$\begin{aligned}
\lambda_t &= \beta E_t \left\{ R_t \frac{P_t}{P_{t+1}} \lambda_{t+1} \right\} \\
\lambda_t^* &= \beta E_t \left\{ R_t^* \frac{P_t^*}{P_{t+1}^*} \lambda_{t+1}^* \right\} \\
\lambda_t &= \Phi \left(\frac{S_t B_t^F}{P_t Y_t} \right) \beta E_t \left\{ R_t^* \frac{Q_{t+1}}{Q_t} \lambda_{t+1} \right\}
\end{aligned}$$

where λ_t is the marginal utility of consumption:

$$\begin{aligned}
\lambda_t &= U_C \left(C_t \right) = \frac{\psi_t}{C_t - bC_{t-1}} \\
\lambda_t^* &= U_C \left(C_t^* \right) = \frac{\psi_t^*}{C_t^* - b^* C_{t-1}^*}
\end{aligned}$$

The labor supply decisions in each country are:

$$\lambda_t \frac{W_t}{P_t} = L_t^{\varphi}$$

$$\lambda_t^* \frac{W_t^*}{P_t^*} = \left(L_t^* \right)^{\varphi^*}$$

where:

$$L_t = L_t^h + L_t^N$$

and

$$L_t^* = L_t^{h^*} + L_t^{N^*}$$

Household demand for final tradable and nontradable goods are given by:

$$
\begin{aligned}
C_t^T &= \gamma_c \left(\frac{P_t^T}{P_t}\right)^{-\varepsilon} C_t, \\
C_t^N &= (1-\gamma_c) \left(\frac{P_t^N}{P_t}\right)^{-\varepsilon} C_t. \\
C_t^{T^*} &= \gamma_c^* \left(\frac{P_t^{T^*}}{P_t^*}\right)^{-\varepsilon^*} C_t^*, \\
C_t^{N^*} &= (1-\gamma_c^*) \left(\frac{P_t^{N^*}}{P_t^*}\right)^{-\varepsilon} C_t^*
\end{aligned}
$$

and the CPI's in each country are given by:

$$
\begin{aligned}
P_t &\equiv \left[\gamma_c \left(P_t^T\right)^{1-\varepsilon} + (1-\gamma_c)\left(P_t^N\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \\
P_t^* &\equiv \left[\gamma_c^* \left(P_t^{T^*}\right)^{1-\varepsilon} + (1-\gamma_c^*)\left(P_t^{N^*}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

The real exchange rate is

$$
Q_t = \frac{S_t P_t^*}{P_t}
$$

A.3.2 Final Goods Producers

The production of final tradable goods in both countries is given by:

$$
Y_t^T = \left\{\gamma_x^{1/\theta}\left(X_t^h\right)^{\frac{\theta-1}{\theta}} + (1-\gamma_x)^{1/\theta}\left(X_t^f\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}
$$

and

$$
Y_t^{T^*} = \left\{(1-\gamma_x)^{1/\theta}\left(X_t^{h^*}\right)^{\frac{\theta-1}{\theta}} + \gamma_x^{1/\theta}\left(X_t^{f^*}\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}
$$

Demand for intermediate tradable goods is:

$$
X_t^h = \gamma_x \left(\frac{P_t^h}{P_t^T}\right)^{-\theta} Y_t^T;\ X_t^{h^*} = (1-\gamma_x)\left(\frac{P_t^{h^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}
$$

$$
X_t^f = (1-\gamma_x)\left(\frac{P_t^f}{P_t^T}\right)^{-\theta} Y_t^T;\ X_t^{f^*}(f) = \gamma_x \left(\frac{P_t^{f^*}}{P_t^{T^*}}\right)^{-\theta} Y_t^{T^*}
$$

where

$$
P_t^h \equiv \left[\int_0^1 P_t^h(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}},\ P_t^f \equiv \left[\int_0^1 P_t^f(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}.
$$

The price of final tradable goods is:

$$P_t^T = \left[\gamma_x \left(P_t^h\right)^{1-\theta} + (1-\gamma_x)\left(P_t^f\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

and

$$P_t^{T^*} = \left[\gamma_x^* \left(P_t^{h^*}\right)^{1-\theta} + (1-\gamma_x^*)\left(P_t^{f^*}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Since we assumed that the law of one price holds for intermediate goods, it also holds in the aggregate, such that $P_t^h = P_t^{h^*} S_t$, and $P_t^f = P_t^{f^*} S_t$, where S_t is the nominal exchange rate.

A.3.3 Nontradable Goods Producers

The price setting equations are given by the following optimal expressions:

$$\frac{\hat{p}_t^N}{P_t^N} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N}\right)^{-\sigma} MC_{t+k}^N Y_{t+k}^N}{\sum_{k=0}^{\infty} \beta^k \alpha_N^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^N\right)^{\varphi_N}}{\Pi_{t+s}^N}\right)^{1-\sigma} \frac{P_{t+k}^N}{P_{t+k}} Y_{t+k}^N} \right\}$$

where

$$MC_t^N = \frac{W_t}{P_t Z_t^N A_t},$$

$$Y_t^N = C_t^N + G_t^N$$

The evolution of the price level of nontradables is

$$P_t^N \equiv \left[\alpha_N \left(P_{t-1}^N \left(\Pi_{t-1}^N\right)^{\varphi_N}\right)^{1-\sigma} + (1-\alpha_N)\left(\hat{p}_t^N\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^N = \frac{P_{t-1}^N}{P_{t-2}^N}$.

The production function is:

$$Y_t^N = A_t Z_t^N L_t^N.$$

In the foreign country these expressions are:

$$\frac{\hat{p}_t^{N^*}}{P_t^{N^*}} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_{N^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}}{\Pi_{t+s}^{N^*}}\right)^{-\sigma} MC_{t+k}^{N^*} Y_{t+k}^{N^*}}{\sum_{k=0}^{\infty} \beta^k \alpha_{N^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{N^*}\right)^{\varphi_{N^*}}}{\Pi_{t+s}^{N^*}}\right)^{1-\sigma} \frac{P_{t+k}^{N^*}}{P_{t+k}} Y_{t+k}^{N^*}} \right\}$$

where

$$MC_t^{N^*} = \frac{W_t}{P_t Z_t^{N^*} A_t},$$

$$Y_t^{N^*} = C_t^{N^*} + G_t^{N^*}$$

The evolution of the price level of nontradables is

$$P_t^{N^*} \equiv \left[\alpha_{N^*} \left(P_{t-1}^{N^*} \left(\Pi_{t-1}^{N^*} \right)^{\varphi_{N^*}} \right)^{1-\sigma} + (1-\alpha_{N^*}) \left(\hat{p}_t^{N^*} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^{N^*} = \frac{P_{t-1}^{N^*}}{P_{t-2}^{N^*}}$.

The production function is:

$$Y_t^{N^*} = A_t Z_t^{N^*} L_t^{N^*}.$$

A.3.4 Intermediate Traded Goods Producers

The price setting equations are given by the following optimal expressions:

$$\frac{\hat{p}_t^h}{P_t^h} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_h^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^h\right)^{\varphi_h}}{\Pi_{t+s}^h} \right)^{-\sigma} MC_{t+k}^h Y_{t+k}^h}{\sum_{k=0}^{\infty} \beta^k \alpha_h^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^h\right)^{\varphi_h}}{\Pi_{t+s}^h} \right)^{1-\sigma} \frac{P_{t+k}^h}{P_{t+k}} Y_{t+k}^h} \right\}$$

where

$$MC_t^h = \frac{W_t}{P_t Z_t^h A_t},$$

$$Y_t^h = X_t^h + X_t^{h^*}.$$

The evolution of the price level of final tradables is

$$P_t^h \equiv \left[\alpha_h \left(P_{t-1}^h \left(\Pi_{t-1}^h \right)^{\varphi_h} \right)^{1-\sigma} + (1-\alpha_h) \left(\hat{p}_t^h \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^h = \frac{P_{t-1}^h}{P_{t-2}^h}$.

The production function is:

$$Y_t^h = A_t Z_t^h L_t^h$$

In the foreign country, this expressions are:

$$\frac{\hat{p}_t^{f^*}}{P_t^{f^*}} = \frac{\sigma}{(\sigma-1)} E_t \left\{ \frac{\sum_{k=0}^{\infty} \beta^k \alpha_{f^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{f^*}\right)^{\varphi_{f^*}}}{\Pi_{t+s}^{f^*}} \right)^{-\sigma} MC_{t+k}^{f^*} Y_{t+k}^{f^*}}{\sum_{k=0}^{\infty} \beta^k \alpha_{f^*}^k \lambda_{t+k} \left(\prod_{s=1}^{k} \frac{\left(\Pi_{t+s-1}^{f^*}\right)^{\varphi_{f^*}}}{\Pi_{t+s}^{f^*}} \right)^{1-\sigma} \frac{P_{t+k}^{f^*}}{P_{t+k}} Y_{t+k}^{f^*}} \right\}$$

where

$$MC_t^{f^*} = \frac{W_t}{P_t Z_t^{f^*} A_t},$$

$$Y_t^{f^*} = X_t^f + X_t^{f^*}.$$

The evolution of the price level of final tradables is

$$P_t^{f^*} \equiv \left[\alpha_{f^*}\left(P_{t-1}^{f^*}\left(\Pi_{t-1}^{f^*}\right)^{\varphi_{f^*}}\right)^{1-\sigma} + (1-\alpha_{f^*})\left(\hat{p}_t^{f^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where $\Pi_{t-1}^{f^*} = \frac{P_{t-1}^{f^*}}{P_{t-2}^{f^*}}$.

The production function is:

$$Y_t^{f^*} = A_t Z_t^{f^*} L_t^{f^*}$$

A.3.5 Monetary Policy

Monetary policy in both countries is conducted with a Taylor rule that targets CPI inflation and output growth deviation from steady-state values:

$$R_t = \bar{R}^{(1-\rho_r)} R_{t-1}^{\rho_r} \left(\frac{P_t/P_{t-1}}{\Pi}\right)^{(1-\rho_r)\gamma_\pi} \left(\frac{Y_t/Y_{t-1}}{1+g}\right)^{(1-\rho_r)\gamma_y} \exp(\varepsilon_t^r)$$

$$R_t^* = \bar{R}^{*(1-\rho_r^*)} \left(R_{t-1}^*\right)^{\rho_r^*} \left(\frac{P_t^*/P_{t-1}^*}{\Pi^*}\right)^{(1-\rho_r^*)\gamma_\pi^*} \left(\frac{Y_t^*/Y_{t-1}^*}{1+g}\right)^{(1-\rho_r^*)\gamma_y^*} \exp(\varepsilon_t^{r^*})$$

A.3.6 Demand Shocks

$$G_t^T = (\bar{G}^T)^{(1-\rho_{G^T})}(G_{t-1}^T)^{\rho_{G^T}} \exp(\varepsilon_t^{G^T})$$

$$G_t^N = (\bar{G}^N)^{(1-\rho_{G^N})}(G_{t-1}^N)^{\rho_{G^N}} \exp(\varepsilon_t^{G^N})$$

$$G_t^{T^*} = (\bar{G}^{T^*})^{(1-\rho_{G^{T*}})}(G_{t-1}^{T^*})^{\rho_{G^{T*}}} \exp(\varepsilon_t^{G^{T^*}})$$

$$G_t^{N^*} = (\bar{G}^{N^*})^{(1-\rho_{G^{N*}})}(G_{t-1}^{N^*})^{\rho_{G^{N*}}} \exp(\varepsilon_t^{G^{N^*}})$$

A.3.7 Trade Balance and Net Foreign Asset Dynamics

We present the evolution of the trade balance and net foreign assets of the home country, since the definition those in the foreign country will mirror those in the home country. Holdings of foreign bonds depend on the trade balance (NX_t) as follows

$$\frac{S_t B_t^F}{P_t R_t^* \Phi\left(\frac{S_t B_t^F}{P_t Y_t}\right)} = \frac{S_t B_{t-1}^F}{P_t} + NX_t$$

Since international trade only occurs at the intermediate goods level, net exports equal exports minus imports of intermediate goods:

$$NX_t = \frac{P_t^h X_t^{h^*} - P_t^f X_t^f}{P_t}$$

Finally, we define nominal GDP to be equal to aggregate nominal private and public consumption, hence $P_t Y_t = P_t^T (C_t^T + G_t^T) + P_t^N (C_t^N + G_t^N)$.

B Appendix: Log-Linear version of the Model

Euler equations

$$b\Delta c_t = -\left(1+g-b\right)\left(r_t - E_t\triangle p_{t+1}\right) + \left(1+g\right)E_t\Delta c_{t+1} + \left(1+g-b\right)\left(1-\rho_\psi\right)\widehat{\psi}_t \tag{19}$$

$$b^*\Delta c_t^* = -\left(1+g-b^*\right)\left(r_t^* - E_t\triangle p_{t+1}^*\right) + \left(1+g\right)E_t\Delta c_{t+1}^* + \left(1+g-b^*\right)\left(1-\rho_\psi^*\right)\widehat{\psi}_t^* \tag{20}$$

Risk sharing

$$\begin{aligned} E_t\left(q_{t+1} - q_t\right) &= \left[\frac{\left(1+g\right)E_t\Delta c_{t+1} - b\Delta c_t}{\left(1+g-b\right)}\right] - \left[\frac{\left(1+g\right)E_t\Delta c_{t+1}^* - b^*\Delta c_t^*}{\left(1+g-b^*\right)}\right] \\ &\quad + \left(1-\rho_\psi\right)\widehat{\psi}_t - \left(1-\rho_\psi^*\right)\widehat{\psi}_t^* + \chi b_t \end{aligned} \tag{21}$$

where $\chi \equiv -\Phi'\left(0\right)Y$, $b_t = \left(\frac{S_t B_t^F}{P_t}\right)Y^{-1}$.

The labor supply schedules are given by:

$$\tilde{\omega}_t = \varphi l_t + \left[\frac{1+g}{1+g-b}\right]\widetilde{c}_t - \frac{b}{\left(1+g-b\right)}\widetilde{c}_{t-1} + \frac{b}{\left(1+g-b\right)}\varepsilon_t^a \tag{22}$$

$$\tilde{\omega}_t^* = \varphi l_t^* + \left[\frac{1+g}{1+g-b^*}\right]\widetilde{c}_t^* - \frac{b^*}{\left(1+g-b^*\right)}\widetilde{c}_{t-1}^* + \frac{b^*}{\left(1+g-b^*\right)}\varepsilon_t^a \tag{23}$$

Technology

$$\widetilde{y}_t^h = l_t^h + z_t^h - \varepsilon_t^a \tag{24}$$

$$\widetilde{y}_t^{h^*} = l_t^{h^*} + z_t^{h^*} - \varepsilon_t^a \tag{25}$$

$$\widetilde{y}_t^N = l_t^N + z_t^N - \varepsilon_t^a \tag{26}$$

$$\widetilde{y}_t^{N^*} = l_t^{N^*} + z_t^{N^*} - \varepsilon_t^a \tag{27}$$

Consumer price inflation

$$\triangle p_t = \gamma_c\triangle p_t^T + \left(1-\gamma_c\right)\triangle p_t^N \tag{28}$$

$$\triangle p_t = \gamma_{c^*}\triangle p_t^{T^*} + \left(1-\gamma_{c^*}\right)\triangle p_t^{N^*} \tag{29}$$

Tradable inflation

$$\triangle p_t^T = \gamma_x\triangle p_t^h + \left(1-\gamma_x\right)\left(\triangle p_t^{f^*} + \Delta s_t\right) \tag{30}$$

$$\triangle p_t^{T^*} = \gamma_{x^*}\left(\triangle p_t^h - \Delta s_t\right) + \left(1-\gamma_{x^*}\right)\triangle p_t^{f^*} \tag{31}$$

Price setting in the nontradable sector

$$\triangle p_t^N - \varphi_N\triangle p_{t-1}^N = \beta E_t\left(\triangle p_{t+1}^N - \varphi_N\triangle p_t^N\right) + \kappa_N\left(\widetilde{w}_t - z_t^N - t_t^N\right) \tag{32}$$

$$\triangle p_t^{N^*} - \varphi_{N^*} \triangle p_{t-1}^{N^*} = \beta E_t \left(\triangle p_{t+1}^{N^*} - \varphi_N^* \triangle p_t^{N^*} \right) + \kappa_{N^*} \left(\widetilde{w}_t^* - z_t^{N^*} - t_t^{N^*} \right) \tag{33}$$

where $\kappa_N = (1 - \alpha_N)(1 - \beta \alpha_N)/\alpha_N$, $\kappa_{N^*} = (1 - \alpha_{N^*})(1 - \beta \alpha_{N^*})/\alpha_{N^*}$, $t^N = p_t^N - p_t$, and $t^{N^*} = p_t^{N^*} - p_t$.

Price setting in the intermediate tradable good sector

$$\triangle p_t^h - \varphi_h \triangle p_{t-1}^h = \beta E_t \left(\triangle p_{t+1}^h - \varphi_h \triangle p_t^h \right) + \kappa_h \left(\widetilde{w}_t - z_t^h - t_t^h - t_t^T \right) \tag{34}$$

$$\triangle p_t^{f^*} - \varphi_{f^*} \triangle p_{t-1}^{f^*} = \beta E_t \left(\triangle p_{t+1}^{f^*} - \varphi_f^* \triangle p_t^{f^*} \right) + \kappa_{f^*} \left(\widetilde{w}_t^* - z_t^{f^*} - t_t^{f^*} - t_t^{T^*} \right) \tag{35}$$

where $\kappa_h = (1 - \alpha_h)(1 - \beta \alpha_h)/\alpha_h$, $\kappa_{f^*} = (1 - \alpha_{f^*})(1 - \beta \alpha_{f^*})/\alpha_{f^*}$, $t_t^h = p_t^h - p_t^T$, $t_t^{f^*} = p_t^{f^*} - p_t^{T^*}$, $t^T = p_t^T - p_t$, and $t^{T^*} = p_t^{T^*} - p_t$.

Final consumption demand

$$\widetilde{c}_t^T = -\varepsilon t_t^T + \widetilde{c}_t \tag{36}$$

$$\widetilde{c}_t^{T^*} = -\varepsilon^* t_t^{T^*} + \widetilde{c}_t^* \tag{37}$$

$$\widetilde{c}_t^N = -\varepsilon t_t^N + \widetilde{c}_t \tag{38}$$

$$\widetilde{c}_t^{N^*} = -\varepsilon^* t_t^{N^*} + \widetilde{c}_t^* \tag{39}$$

Intermediate tradable and nontradable demand

$$\widetilde{x}_t^h = -\theta t_t^h + \widetilde{y}_t^T \tag{40}$$

$$\widetilde{x}_t^{h^*} = -\theta t_t^{h^*} + \widetilde{y}_t^{T^*} \tag{41}$$

$$\widetilde{x}_t^f = -\theta t_t^f + \widetilde{y}_t^T \tag{42}$$

$$\widetilde{x}_t^{f^*} = -\theta t_t^{f^*} + \widetilde{y}_t^{T^*} \tag{43}$$

Relative Price Index

Let´s define $t_t = \frac{p_t^f}{p_t^h}$, since the law of one price holds $t_t = -t_t^* = \frac{p_t^{h^*}}{p_t^{f^*}}$, then we can write the following relative prices as a function of t_t.

$$\begin{aligned} t_t^h &= -(1 - \gamma_x) t_t && (44) \\ t_t^{h^*} &= -\gamma_x t_t && (45) \\ t_t^f &= \gamma_x t_t && (46) \\ t_t^{f^*} &= (1 - \gamma_x) t_t && (47) \end{aligned}$$

Relative prices

$$t_t = t_{t-1} + \triangle s_t + \triangle p_t^{f^*} - \triangle p_t^h \tag{48}$$

$$t_t^T = t_{t-1}^T + \triangle p_t^T - \triangle p_t \tag{49}$$

$$t_t^N = -\frac{\gamma_c}{1-\gamma_c} t_t^T \tag{50}$$

$$t_t^{T^*} = t_{t-1}^{T^*} + \triangle p_t^{T^*} - \triangle p_t^* \tag{51}$$

$$t_t^{N^*} = -\frac{\gamma_c^*}{1-\gamma_c^*} t_t^{T^*} \tag{52}$$

$$q_t = q_{t-1} + \triangle s_t + \triangle p_t^* - \triangle p_t \tag{53}$$

Taylor rules

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\,\gamma_\pi \triangle p_t + \gamma_y \triangle y_t + \varepsilon_t^r \tag{54}$$

$$r_t^* = \rho_r^* r_{t-1}^* + (1-\rho_{r^*})\,\gamma_\pi^* \triangle p_t^* + \gamma_y^* \triangle y_t + \varepsilon_t^{r^*} \tag{55}$$

Net foreign assets and net exports

$$\beta \widetilde{b}_t - \frac{1}{1+g}\widetilde{b}_{t-1} = \widetilde{nx}_t \tag{56}$$

$$\widetilde{nx}_t = \frac{X^f}{Y}\left(\widetilde{x}_t^{h^*} - \widetilde{x}_t^f - t_t\right) \tag{57}$$

where $\widetilde{b}_t = \frac{\bar{B}_t^F S_t}{P_t Y}$ is the debt to GDP ratio, and $\widetilde{nx}_t = \frac{NX_t}{Y}$, and where we have assumed balanced trade in the steady state. To solve for the steady-state ratios, we have that:

$$\frac{Y^T}{Y} = \frac{G^T}{G} = \frac{C^T}{C} = \gamma_c$$

$$\frac{Y^N}{Y} = \frac{G^N}{G} = \frac{C^N}{C} = 1 - \gamma_c$$

$$\frac{X^h}{Y^T} = \gamma_x, \ \frac{X^f}{Y^T} = 1 - \gamma_x$$

Therefore tradable GDP over total GDP is

$$\frac{X^h}{Y} = \frac{X^h}{Y^T}\frac{Y^T}{Y} = \gamma_x \gamma_c.$$

Hence

$$\frac{X^f}{Y} = \frac{X^f}{X^T}\frac{X^T}{Y^T}\frac{Y^T}{Y} = (1-\gamma_x)\gamma_c.$$

Market clearing:

$$\tilde{y}_t^T = (1-\gamma)\tilde{c}_t^T + \gamma \tilde{g}_t^T \tag{58}$$

where γ equals the fraction of government spending over total output. For the nontradable good, the market clearing condition is:

$$\tilde{y}_t^N = (1-\gamma)\tilde{c}_t^N + \gamma\tilde{g}_t^N \tag{59}$$

Finally, for the intermediate tradable goods sector:

$$\widetilde{y}_t^h = \gamma_x \widetilde{x}_t^h + (1-\gamma_x)\,\widetilde{x}_t^{h^*} \tag{60}$$

Total real GDP

$$\widetilde{y}_t = \gamma_c(t_t^T + \widetilde{y}_t^T) + (1-\gamma_c)\,(t_t^N + \widetilde{y}_t^N) \tag{61}$$

total labor

$$l_t = \frac{X^h}{X^h + Y^N} l_t^h + \frac{Y^N}{X^h + Y^N} l_t^N \tag{62}$$

For the foreign country:

$$\tilde{y}_t^{T^*} = (1-\gamma^*)\tilde{c}_t^{T^*} + \gamma^* \tilde{g}_t^{T^*} \tag{63}$$

$$\widetilde{y}_t^{N^*} = (1-\gamma^*)\widetilde{c}_t^{N^*} + \gamma^* \widetilde{g}_t^{N^*} \tag{64}$$

$$\widetilde{y}_t^{f^*} = \gamma_x^* \widetilde{x}_t^{f^*} + (1-\gamma_x^*)\,\widetilde{x}_t^f \tag{65}$$

$$\widetilde{y}_t^* = \gamma_c^*(t_t^{T^*} + \widetilde{y}_t^{T^*}) + (1-\gamma_c^*)\,(t_t^{N^*} + \widetilde{y}_t^{N^*}) \tag{66}$$

$$l_t = \frac{Y^{f^*}}{Y^h + Y^N} l_t^{f^*} + \frac{Y^{N^*}}{Y^{f^*} + Y^{N^*}} l_t^{N*} \tag{67}$$

Mapping variables in the model with observable variables.

$$\begin{aligned} \widetilde{c}_t - \widetilde{c}_{t-1} &= \triangle c_t - \varepsilon_t^a && (68) \\ \widetilde{c}_t^* - \widetilde{c}_{t-1}^* &= \triangle c_t^* - \varepsilon_t^a && (69) \\ \widetilde{y}_t - \widetilde{y}_{t-1} &= \triangle y_t - \varepsilon_t^a && (70) \\ \widetilde{y}_t^* - \widetilde{y}_{t-1}^* &= \triangle y_t^* - \varepsilon_t^a && (71) \end{aligned}$$

Shocks

$$\psi_t = \rho_\psi \psi_{t-1} + \varepsilon_t^\psi$$

$$\psi_t^* = \rho_{\psi^*} \psi_{t-1}^* + \varepsilon_t^{\psi^*}$$

$$z_t^h = \rho^{Z,h} z_{t-1}^h + \varepsilon_t^{Z,h}$$

$$z_t^{f^*} = \rho^{Z,f^*} z_{t-1}^{f^*} + \varepsilon_t^{Z,f^*}$$

$$z_t^N = \rho^{Z,N} z_{t-1}^N + \varepsilon_t^{Z,N}$$

$$z_t^{N^*} = \rho^{Z,N^*} z_{t-1}^{N^*} + \varepsilon_t^{Z,N^*}$$

$$g_t^T = \rho^{G,T} g_{t-1}^T + \varepsilon_t^{G,T}$$

$$g_t^N = \rho^{G,N} g_{t-1}^N + \varepsilon_t^{G,N}$$

$$g_t^{T^*} = \rho^{G,T^*} g_{t-1}^{T^*} + \varepsilon_t^{G,T^*}$$

$$g_t^{N^*} = \rho^{G,N^*} g_{t-1}^{N^*} + \varepsilon_t^{G,N^*}$$

and $\varepsilon_t^a, \varepsilon_t^r, \varepsilon_t^{r^*}$ are iid shocks.